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XXVIII.—OBLIQUITY CORRECTIONS IN RADIUM ESTIMATION.

By IVOR BACKHURST, *M.Sc.*, The National Physical Laboratory.

Received February 12, 1926.

ABSTRACT.

Formulæ are given for obliquity corrections in radium estimation applicable to sources of the shapes most frequently occurring in practice. The effect of scattered radiation is considered and some experimental results are shown graphically and compared with theory. Calculated values for a particular case are given by way of example.

WHEN the γ-ray method is used to determine the radium content of a tube having dimensions considerably larger than the radium standard with which comparison is to be made, it may be necessary to apply a correction to allow for the difference in distribution of the two sources. In the case of a long tube placed perpendicular to the axis of the ionization chamber, the parts of the source are at a greater average distance from the latter than in the case of a short tube placed in the same position, and, on account of greater average obliquity, the rays from the longer tube are absorbed to a somewhat larger extent in the lead front of the ionization chamber. The case arises, for example, in the determination of the radium content of a luminous compound. The amount of radium element normally contained in a tube 12 cms. long and 2 cms. diameter filled with compound constitutes a comparatively weak source, so that when 0·5 cm. of lead is used on the front of the ionization chamber, it becomes desirable to effect the determination with the radium distant not more than about 25 or 30 cms. from the latter. Under these conditions the correction for obliquity may amount to 2·5 per cent. A further correction should be applied to allow for absorption of the γ-rays in the compound itself.*

The obliquity correction may be complicated by the effect of scattered radiation if the radium bench is near a wall, or if a substantial stand is used as a holder for the radium.

POINT SOURCE.

The ionization produced by the direct action of the γ-rays on the air in the ionization chamber may be assumed to be small compared with that caused by the electronic radiation excited in the lead walls of the chamber or in metal parts inside it. If the chamber contains only a thin brass electrode, most of the ionization will be due to radiation from the lead walls.

If it is further assumed that the ionizing power of the electronic radiation from a particular area of metal surface inside the chamber is directly proportional to the intensity of the γ-radiation crossing that surface, but independent of the direction of the γ-radiation, then, since absorption of the γ-rays in the air inside the ionization chamber may be neglected, we have in the case of a point source, assuming to begin with that absorption in the lead walls of the chamber remains constant, that the ionization effect " E " is proportional to the solid angle " w " of the tangent cone drawn from the point source to the inner surface of the ionization chamber. In the case of a cylindrical ionization chamber of internal radius " a," the solid angle " w " is for any point distant from the axis of the chamber produced less than " a," the same as the angle subtended by the circular front of the ionization chamber at that point.

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* "The Estimation of the Radium Content of Radioactive Luminous Compounds," by Owen and Fage, Proc. Phys. Soc., Vol. 34, I, pp. 27-32 (December, 1921).

The solid angle subtended by a circle at any point may be expressed with the aid of elliptic functions, or alternatively by means of zonal harmonics. The former method is normally not so convenient as the latter, as in most practical cases the zonal harmonic series converges sufficiently rapidly. If the radius of the circular front of the ionization chamber, denoted by " a " is 7·25 cms., and " d " the distance of the point P from the chamber measured along the axis is 25 cms., we have provided that " y " the distance of P from the axis is not greater than "a," that

$$w=2\pi\left\{\frac{1}{2}p^2 . P_1(c)-\frac{3}{8}p^4 . P_3(c)+\frac{5}{16}p^6 . P_5(c)\right\} \quad . \; . \; . \; . \; . \quad (1)$$

to an accuracy of about 1/10,000.

In the above expression $p=\frac{ac}{d}$, where $c=\cos\theta$, and $\tan\theta=\frac{y}{d}=T$.

$$P_1(c)=c.$$

$$P_3(c)=\tfrac{1}{2}(5c^3-3c)$$

$$P_5(c)=\tfrac{1}{8}(63c^5-70c^3+15c)$$

UNIFORM LINE SOURCE.

In order to find the effect of a line source cutting the axis perpendicularly, it is convenient to express (1) in terms of powers of $\tan\theta$ or y. Writing (1) in powers of c, we have

$$w/2\pi=\frac{1}{2}k^2c^3+\frac{9}{16}k^4c^5+\left(-\frac{15}{16}k^4+\frac{75}{128}k^6\right)c^7-\frac{350}{128}k^6c^9+\frac{315}{128}k^6c^{11}$$

where $k=a/d$. Since T is not greater than 0·29, we may put $c=1-\frac{1}{2}T^2+\frac{3}{8}T^4$, with an error of less than 1/20,000. This gives

$$w/2\pi=\text{Vers}\ \varphi-LT^2+MT^4-NT^6 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where $L=3K^2-30K^4+132K^6$

$M=3{\cdot}75K^2-80K^4$

$N=2{\cdot}5K^2$

and $K=k/2=a/2d$

and $\tan\varphi=k=a/d$

Terms smaller than k^2T^6 are omitted.

The last term of (2) will in most cases be negligible.

Assuming the ionization effect to be proportional to w, we have on integrating (2) with respect to "y," since $y=Td$, that

$$E \propto d\{\text{Vers}\ \varphi(T_2-T_1)-L(T_2^3-T_1^3)/3+M(T_2^5-T_1^5)/5-N(T_2^7-T_1^7)/7\}$$

where E is the effect produced by a uniform line source cutting the axis perpendicularly and extending from $y=y_1$ to $y=y_2$, so that $T_1=y_1/d$ and $T_2=y_2/d$.

In the case of a narrow tube "l" cms. long, bisected by the axis, the percentage correction to be applied to the measured value, in order to obtain the effect that would be produced by the same quantity of radium concentrated on the axis at the middle of the tube, will therefore be given by

$$\frac{100\{LT^2/3-MT^4/5+NT^6/7\}}{\text{Vers}\ \varphi-\{LT^2/3-MT^4/5+NT^6/7\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $T=l/2d$.

FORMULÆ ALLOWING FOR THE CHANGE IN ABSORPTION IN THE LEAD FRONT OF THE IONIZATION CHAMBER DUE TO OBLIQUITY.

If the radium is not brought nearer than about 25 cms. to an ionization chamber of diameter about 14 cms., and is nowhere more than 7 cms. from the axis, an expression for the ionization effect may be found that is sufficiently accurate for most purposes, and that takes into account the change in absorption in the lead front of the ionization chamber due to obliquity.

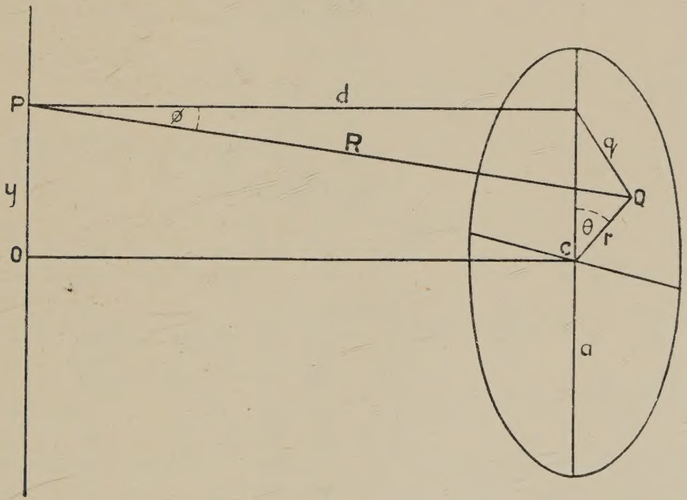

FIG. 1.

POINT SOURCE.

In Fig. 1 let PQ represent a ray from a source at P cutting the inner surface of the front of the ionization chamber at Q, the axis of the chamber lying along OC. Then from the figure

$$R^2=d^2+q^2=d^2+r^2-2ry\cos\theta+y^2$$

If μ is the absorption coefficient in the lead front of thickness t, then if E is the total intensity of γ-radiation inside the lead front due to a point source of unit intensity at P,

$$E=K'\int_0^{2\pi}\int_0^a \frac{\cos\varphi\,.\,e^{-\mu t/\cos\varphi}\,.\,r\,.\,d\theta\,.\,dr.}{R^2}$$

Using the substitutions $R^2=d^2(1+k^2)$

$$k^2=(r^2-2ry\cos\theta+y^2)/d^2$$

$$m=\mu t/2$$

$$K=K'e^{-\mu t}:/d^2$$

we have approximately

$$E=K\int_0^{2\pi}\int_0^a \frac{1-mk^2+\left(\frac{m}{4}+\frac{m^2}{2}\right)k^4}{(1+k^2)^{3/2}}\,.\,r\,.\,d\theta\,.\,dr.$$

Error $\leqslant 0{\cdot}1$ per cent. if $a\leqslant 7$ cms. and $d\geqslant 25$ cms.

And writing $n=\left(\frac{m}{4}+\frac{m^2}{2}\right)$

$$f=3/2+m$$

$$g=\frac{15}{8}+\frac{3m}{2}+n$$

$$h=\frac{35}{16}+\frac{15m}{8}+\frac{3n}{2}$$

then $E=2K\int_0^{\pi}\int_0^a \{1-f\,.\,k^2+g\,.\,k^4-h\,.\,k^6\}\,r\,.\,d\theta\,.\,dr.$

Putting $T=y/d$

$$A=1-\left(\frac{f}{2}\cdot\frac{a^2}{d^2}-\frac{g}{3}\cdot\frac{a^4}{d^4}+\frac{h}{4}\cdot\frac{a^6}{d^6}\right)$$

$$B=f-2g\,.\,\frac{a^2}{d^2}+3h\,.\,\frac{a^4}{d^4}$$

$$C=g-\frac{9h}{2}\cdot\frac{a^2}{d^2}$$

$$D=h$$

we have $E=\pi a^2K\{A-BT^2+CT^4-DT^6\}$ . . . (4)

In the case considered

$a=7{\cdot}0$, $d=25{\cdot}0$, $\mu=0{\cdot}5$ and $t=0{\cdot}5$,

so that

$$E=\pi a^2K\{0{\cdot}94097-1{\cdot}3003T^2+1{\cdot}4766T^4-2{\cdot}4805T^6\}$$

For a value of y as large as 6·5, $2{\cdot}4805T^6=0{\cdot}00077$, so that the last term may usually be neglected with an error less than 0·1 per cent.

UNIFORM LINE SOURCE.

The expression may readily be integrated to obtain the correction for a uniform line source along *OP* (Fig. 1). In the case when the line source is bisected by *OC*, the percentage correction will be found to be

$$\frac{100\{BT^2/3-CT^4/5\}}{A-\{BT^2/3-CT^4/5\}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

UNIFORM RECTANGULAR LAMINAR SOURCE PERPENDICULAR TO THE AXIS *OC*.

From (4) we have that

$$E=\pi a^2K\{A-B'r^2+C'r^4\}$$

where

$$B'=B/d^2$$

$$C'=C/d^4$$

and *r* is the perpendicular distance of a point source of unit intensity from the axis *OC*. The last term of (4) is omitted as being usually negligible.

Taking rectangular co-ordinates *x*, *y* in a plane perpendicular to *OC*, and substituting $x^2+y^2=r^2$, we have as an expression for the total effect of a rectangular source of uniform intensity *I* per unit area

$$\pi a^2KI\int_{x_1}^{x_2}\int_{y_1}^{y_2}\{A-B'(x^2+y^2)+C'(x^2+y^2)^2\}\,.\,dx\,.\,dy.$$

which gives the percentage correction

$$\frac{100(B'R-C'S)}{AQ-(B'R-C'S)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

where

$$Q=(x_2-x_1)(y_2-y_1)$$

$$R=\frac{1}{3}\{(x_2^3-x_1^3)(y_2-y_1)+(y_2^3-y_1^3)(x_2-x_1)\}$$

$$S=\left\{\frac{(x_2^5-x_1^5)}{5}(y_2-y_1)+\frac{2}{9}(x_2^3-x_1^3)(y_2^3-y_1^3)+\frac{(y_2^5-y_1^5)}{5}(x_2-x_1)\right\}$$

In most practical cases the centre of the rectangular laminar will lie on *OC*, so that $x_1=-x_2$ and $y_1=-y_2$, and the expressions for *Q*, *R* and *S* will be considerably shortened.

UNIFORM SQUARE LAMINAR SOURCE CENTRALLY PLACED PERPENDICULAR TO THE AXIS *OC*.

In this case, putting $x_1=y_1=0$, and $x_2=y_2=b$, the side of the square being $2b$, expression (6) for the percentage correction becomes

$$\frac{100\left\{\frac{2}{3}B'\,.\,b^2-\frac{28}{45}C'\,.\,b^4\right\}}{\mathrm{A}-\left\{\frac{2}{3}B'\,.\,b^2-\frac{28}{45}C'\,.\,b^4\right\}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6a)$$

Uniform Circular Disc Source with Centre Distant C from the Axis OC, and in a Plane Perpendicular to OC.

The effect due to a point source of unit intensity distant p from the axis is

$$E=\pi a^2K\{A-B'p^2+C'p^4\}$$

so that if I is the intensity of the source per unit area, we have for the total effect of the disc source, taking polar co-ordinates about the centre of the disc, the expression

$$\pi a^2KI\int_0^{2\pi}\int_0^{R}\{A-B'p^2+C'p^4\}r\,.\,d\theta\,.\,dr.$$

where $p^2=c^2+r^2-2cr\cos\theta$, and R is the radius of the disc, which gives as the percentage correction

$$\frac{100(B'U-C'V)}{A-(B'U-C'V)}\quad\ldots\ldots\ldots\ldots\quad(7)$$

where

$$U=c^2+R^2/2$$

$$V=c^4+2c^2R^2+R^4/3$$

If the centre of the disc lies on OC the percentage correction becomes

$$\frac{100(B'\,.\,R^2/2-C'\,.\,R^4/3)}{A-(B'\,.\,R^2/2-C'\,.\,R^4/3)}\quad\ldots\ldots\ldots\ldots\quad(8)$$

Uniform Cylindrical Source, the Axis of the Cylinder Cutting OC at Right Angles.

If I is the intensity per unit volume of the cylindrical source, then from (4) the total ionization effect produced by the source is given by

$$2\int_0^{\pi}\int_0^{B}\int_0^{L}\left\{\pi a^2KI\left(A-B\,.\,\frac{c^2}{d^2}+C\,.\,\frac{c^4}{d^4}\right)\right\}r\,.\,d\theta\,.\,dr\,.\,dy.\quad\ldots\ldots\quad(9)$$

where, referring to Fig. 2, c is the perpendicular distance of the volume element at W from OC, the term involving D being regarded as negligible.

From the figure we have

$$d=d_0-z\,;\quad z=r\cos\theta$$

$$c^2=y^2+x^2=y^2+r^2-z^2$$

Substituting for c in terms of y, r and $\cos\theta$, and using the expansions of reciprocal powers of d in terms of d_0 and $r\cos\theta$, expression (9) may be integrated to the required degree of approximation. Terms have been neglected which on integration affect the accuracy of the expression to about 0·1 per cent. for the case when $a=7{\cdot}25$, $d=25$, $\mu=0{\cdot}5$, $t=0{\cdot}5$ and $R\not>3$. The percentage correction may then be expressed by

$$\frac{100\{B_0''\,.\,T_0^2/3-C_0''\,.\,T_0^4/5\}}{A_0''-\{B_0''\,.\,T_0^2/3-C_0''\,.\,T_0^4/5\}}\quad\ldots\ldots\ldots\quad(10)$$

where

$$A_0''=A_0-\frac{R^2}{4d_0^2}\left\{f+\frac{3fa^2}{2d_0^2}-\frac{2ga^2}{d_0^2}\right\}$$

$$B_0''=B_0+\frac{R^2}{2d_0^2}\left\{\frac{3f}{2}-g\right\}$$

$$C_0''=C_0$$

The suffix "$_0$" to A, B and C indicates that in their values given in (4) d_0 is to be understood for d.

If $R \not> 1$, the values of a, d, μ and t being the same as before, sufficient accuracy may be obtained by putting

$$A_0''=A_0-\frac{fR^2}{4d_0^2}$$

$$B_0''=B_0$$

$$C_0''=C_0$$

SHORT UNIFORM CYLINDRICAL SOURCE, THE AXIS COINCIDING WITH *OC*.

If the intensity of the source is I per unit volume, we have from (8) that the total ionization effect of the source is proportional to

$$\int_0^L \{A-B'\,.\,R^2/2+C'\,.\,R^4/3\}dz$$

where L is the length of the cylinder and R its radius. Substituting $d=d_0-z$ in the expressions for A, B' and C', and using expansions for reciprocal powers of d as before, the expression obtained for the percentage correction is of the form

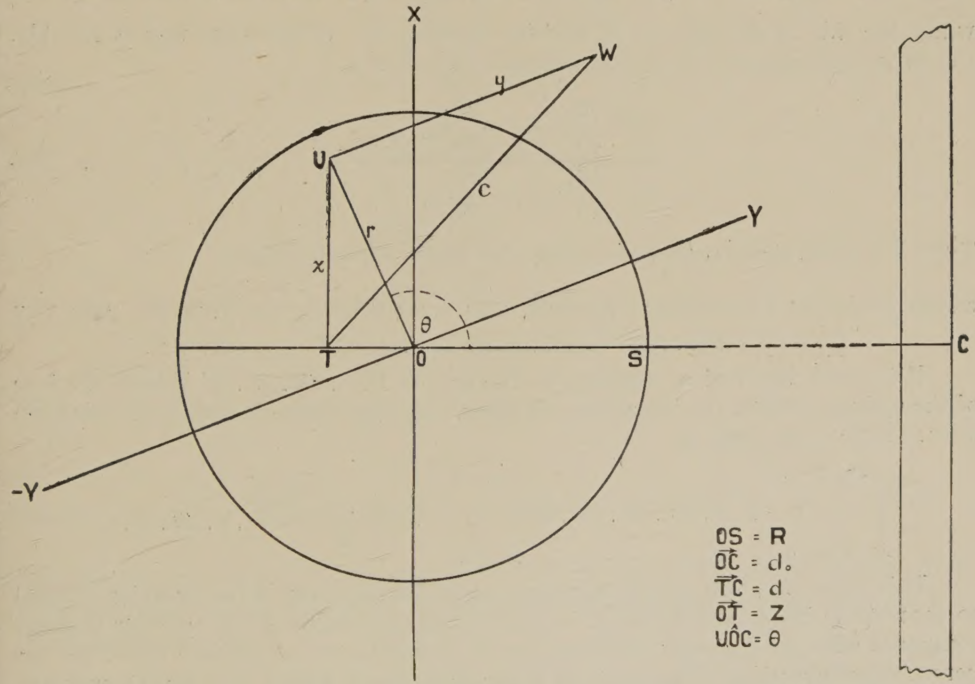

FIG. 2.

$$\frac{100\{B_L\,.\,T_0^2/2-C_L\,.\,T_0^4/3\}}{A_L-\{B_L\,.\,T_0^2/2-C_L\,.\,T_0^4/3\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where $T_0=R/d_0$.

If L is not greater than 0·5 and R is not greater than " a," the values of " a," d, μ and t being the same as before, we have with sufficient accuracy—i.e., 0·1 per cent.,

$$A_L=1-\left\{\frac{f\,.\,a^2}{d_0^{\,2}}\left(1+\frac{L}{d_0}\right)-\frac{g\,.\,a^4}{3d_0^{\,4}}+\frac{ha^6}{8d_0^{\,6}}\right\}$$

$$B_L=\left\{f\left(1+\frac{L}{d_0}\right)-\frac{2ga^2}{d_0^{\,2}}+\frac{3ha^4}{d_0^{\,4}}\right\}$$

$$C_L=g-\frac{9h}{2}\cdot\frac{a^2}{d_0^{\,2}}$$

UNIFORM SOURCE IN THE FORM OF A SHORT PRISM OF SQUARE SECTION, THE AXIS COINCIDING WITH OC.

We have from (6a) that the total ionization effect of the source is proportional to

$$\int_0^L\left\{A-\frac{2}{3}B'b^2+\frac{28}{45}C'b^4\right\}dz$$

where the side of the prism is $2b$ and its length is L. In the same way as for (11), the expression obtained for the percentage correction is

$$\frac{100\left\{\frac{2}{3}B_LT_0^{\,2}-\frac{28}{45}C_LT_0^{\,4}\right\}}{A_L-\left\{\frac{2}{3}B_LT_0^{\,2}-\frac{28}{45}C_LT_0^{\,4}\right\}}\quad\cdots\cdots\cdots\quad(12)$$

where $T_0=b/d_0$ and A_L, B_L and C_L have the same values as before.

SHORT UNIFORM CYLINDRICAL SOURCE, THE AXIS COINCIDING WITH OC, AND THE LINEAR ABSORPTION COEFFICIENT IN THE CYLINDER BEING ν'.

If t' is the depth of an element of the source $I\,.\,r\,.\,d\theta\,.\,dr\,.\,dt'$ behind the face of the cylinder nearer the ionization chamber, the expression for the total ionization effect of the source will be

$$\int_0^{2\pi}\int_0^{R}\int_0^{Z} I\,.\,r\,.\,d\theta\,.\,dr\,.\,dt'\,.\,K'\int_0^{2\pi}\int_0^{a}\frac{\cos\varphi\,.\,e^{-\frac{\mu t+\nu' t'}{\cos\varphi}}}{R^2}\,.\,r\,.\,d\theta\,.\,dr.$$

This expression will only be accurate when the diameter of the cylinder is equal to that of the ionization chamber, but providing the length of the cylinder is small compared with its diameter the error involved will have a negligible effect on the percentage correction. The value of ν' for ZnS, BaCl, $BaSO_4$ and $BaCO_3$ will not exceed 0·1 cm.$^{-1}$, so that in these cases we have with sufficient accuracy

$$\int_0^{2\pi}\int_0^{R} I\,.\,r\,.\,d\theta\,.\,dr\,.\,K'\int_0^{2\pi}\int_0^{a}\frac{\cos\,.\,\varphi\,.\,e^{-\frac{\mu t+\nu' t'}{\cos\varphi}}}{R^2}\,.\,r\,.\,d\theta\,.\,dr.$$

$$=\pi^2R^2a^2KI\left\{A'''-B'''\,.\,\frac{R^2}{2}+C'''\,.\,\frac{R^4}{3}\right\}.\quad\cdots\quad\text{from}\quad(8)$$

where
$$A'''=1-\left\{\frac{f'.a^2}{2(d+t')^2}-\frac{g'a^4}{3(d+t')^4}+\frac{h'a^6}{4(d+t')^6}\right\}$$
$$B'''=\frac{f'}{(d+t')^2}-\frac{2g'a^2}{(d+t')^4}+\frac{3h'a^4}{(d+t')^6}$$
$$C'''=\frac{g'}{(d+t')^4}-\frac{9h'a^2}{2(d+t')^6}$$

and
$$f'=f+v't'$$
$$g'=g+\frac{7v't'}{4}$$
$$h'=h+\frac{9v't'}{4}$$

Integrating with regard to t' from $t'=o$ to $t'=z$, we obtain for the percentage correction
$$\frac{100\,.\,\{B_zT^2/2-C_zT^4/3\}}{A_z-\{B_zT^2/2-C_zT^4/3\}}. \quad \ldots\ldots\ldots \quad (13)$$

where
$$A_z=1-\left(\frac{f_za^2}{2d^2}-\frac{g_za^4}{3d^4}+\frac{h_za^6}{4d^6}\right)$$
$$B_z=f_z-\frac{2g_za^2}{d^2}+\frac{3h_za^4}{d^4}$$
$$C_z=g_z-\frac{9h_za^2}{2d^2}$$
$$f_z=f\left(1-\frac{z}{d}\right)+\frac{v'z}{2}$$
$$g_z=g\left(1-\frac{2z}{d}\right)+\frac{7v'z}{8}$$
$$h_z=h$$

and
$$\mathrm{T}=R/d$$

assuming as before that the length of the cylinder, here denoted by z, is $\ngtr$ 3·5 cm.

UNIFORM SOURCE IN THE FORM OF A SHORT PRISM OF SQUARE SECTION, THE AXIS COINCIDING WITH OC, AND THE LINEAR ABSORPTION COEFFICIENT IN THE PRISM BEING v'.

In the same way as (13) is obtained from (8) we obtain from (6a) for the percentage correction
$$\frac{100\left\{\frac{2}{3}B_z\,.\,\mathrm{T}^2-\frac{28}{45}C_z\,.\,\mathrm{T}^4\right\}}{A_z-\left\{\frac{2}{3}B_z\,.\,\mathrm{T}^2-\frac{28}{45}C_z\,.\,\mathrm{T}^4\right\}}. \quad \ldots\ldots\ldots \quad (14)$$

where A_z, B_z and C_z have the same values as in (13).

CORRECTION FOR THE EFFECT OF SCATTERED RADIATION.

The measured ionization effect E'_y, due to a point source distant y from the axis, will consist of two parts, E_y due to direct radiation and E_s due to radiation scattered from surrounding objects. If the materials producing the greater part of the scattering are at a distance from the source, large compared with y, then, to a first approximation, E_s will be independent of y and constant for a given distance from the ionization chamber.

We have therefore

$$E_y'=E_y+E_s \text{ and } E_0'=E_0+E_s$$

and

$$\frac{E_0'-E_y'}{E_y'}=\frac{E_0-E_y}{E_y}\times\frac{1}{1+E_s/E_y}$$

If

$$\frac{E_0'-E_y'}{E_y'}=c' \text{ and } \frac{E_0-E_y}{E_y}=c$$

we have

$$\frac{E_s}{E_y}=\frac{c-c'}{c'} \qquad (15)$$

where $100c'$ is the percentage correction determined experimentally for a point source distant y from the axis, and $100c$ is the percentage correction as calculated. In this way an estimation of the amount of scattered radiation may be readily made.

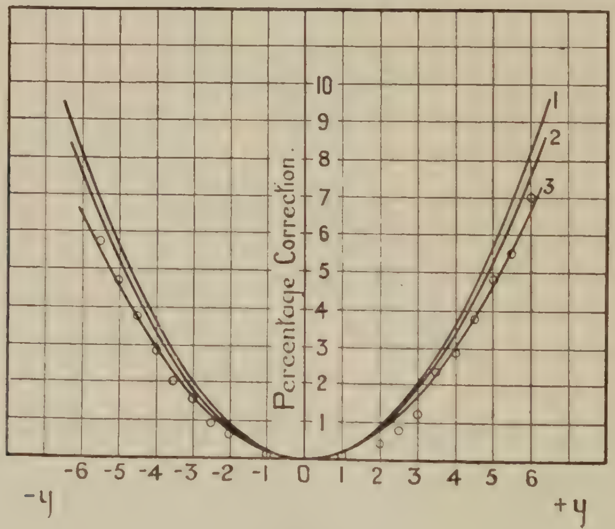

FIG. 3.

Since E_s is constant, we may put $E_s=K_0 \,.\, E_0$, so that

$$c'=\frac{c}{1+K_0.E_0/E_y} \qquad (16)$$

and

$$K_0=\frac{c-c'}{c'(c+1)}. \qquad (17)$$

In Fig. 3 the percentage correction is shown for different values of y for the case of a point source when $a=7{\cdot}0$, $d=25$, $\mu=0{\cdot}5$, $t=0{\cdot}5$.

Curve 1 is obtained from

$$100c=\frac{100\{BT^2-CT^4\}}{A-\{BT^2-CT^4\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{from (4)}$$

Curve 2 is obtained from

$$100c=\frac{100\{LT^2-MT^4\}}{\text{Vers }\varphi-\{LT^2-MT^4\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{from (2)}$$

Curve 3 is obtained from Curve 1 using the relation

$$100c'=\frac{100c}{1+K_0\cdot\frac{E_0}{E_y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{from (16)}$$

with $K_0=0{\cdot}21$.

The experimental points indicated were obtained using as source a small thin-walled glass tube containing radium sulphate. In the case of sulphate the greater part of the active deposit appears to reside on the sulphate powder, and only a small part on the glass walls of the tube, so that with the sulphate shaken to one end of the tube an approximation to a point source could be obtained. With a view to reducing the effect of scattered radiation, the experiment was carried out in the centre of the room, a light table being used to support the apparatus and the ionization chamber and radium tube were supported rather more than twelve inches above the table. The holder for the radium tube was a strip of thin stiff paper about twelve inches long folded to a V-section and supported at the ends by two light wooden rods.

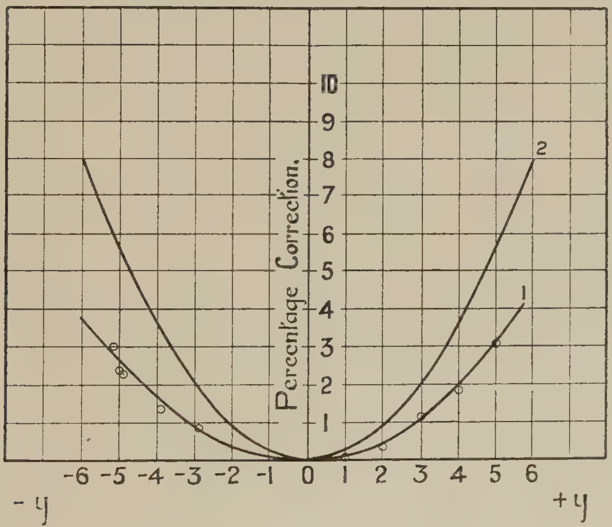

FIG. 4.

In Fig. 4 the percentage correction is shown for different values of y for the case of a point source when $a=7\cdot25$, $d=25$, $\mu=0\cdot5$, $t=0\cdot5$.

Curve 1 is drawn through the experimental points.

Curve 2 is calculated from

$$100c=\frac{100\{BT^2-CT^4\}}{A-\{BT^2-CT^4\}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{from (4)}$$

where $A=0\cdot93721$, $B=1\cdot2821$, $C=1\cdot4130$.

In this case the experiment was carried out on a radium bench parallel and close to one of the walls of the room, a wooden holder being used to support the radium tube. Negative values of y indicate positions on the wall side of the axis OC. The effect of scattered radiation is considerably more marked than in Fig. 3, and the proximity of the wall is sufficient to cause some asymmetry of the experimental curve.

In Table I the percentage correction is shown for different values of d with tubes of different lengths, placed centrally and perpendicular to OC, for the case when $a=7\cdot25$, $\mu=0\cdot5$ and $t=0\cdot5$. The values (which are calculated from 10) are for tubes having diameters not greater than 2 cms., no allowance being made for the effect of scattered radiation.

TABLE I.—*Percentage Correction.*

Diameter of tube not greater than 2 cms.

Distance in cms. (d)	Length of tube in cms.									
	12	11	10	9	8	7	6	5	4	3
25	2·6	2·2	1·8	1·5	1·2	0·9	0·6	0·4	0·3	0·2
30	1·9	1·6	1·3	1·1	0·8	0·7	0·5	0·3	0·2	0·1
35	1·4	1·2	1·0	0·8	0·6	0·5	0·4	0·3	0·2	
40	1·1	0·9	0·8	0·6	0·5	0·4	0·3	0·2	0·1	
45	0·9	0·8	0·6	0·5	0·4	0·3	0·2	0·2	0·1	
50	0·7	0·6	0·5	0·4	0·3	0·3	0·2	0·1		
55	0·6	0·5	0·4	0·3	0·3	0·2	0·2	0·1		
60	0·5	0·4	0·4	0·3	0·2	0·2	0·1			
65	0·4	0·4	0·3	0·2	0·2	0·1	0·1			
70	0·4	0·3	0·3	0·2	0·2	0·1				
75	0·3	0·3	0·2	0·2	0·1	0·1				
80	0·3	0·2	0·2	0·2	0·1					
85	0·3	0·2	0·2	0·1	0·1					
90	0·2	0·2	0·2	0·1	0·1					
95	0·2	0·2	0·1	0·1						
100	0·2	0·2	0·1	0·1						
110	0·2	0·1	0·1							
120	0·1	0·1								
130	0·1									

In conclusion, the author wishes to thank Dr. E. A. Owen for his kind advice and interest in this work.

SUMMARY.

Formulæ are given for obliquity corrections in radium estimation applicable in the following cases :—

1. Point source.
2. Uniform line source.
3. Uniform rectangular laminar source perpendicular to OC.
4. Uniform square laminar source centrally placed perpendicular to OC.
5. Uniform circular disc source in a plane perpendicular to OC.
6. Uniform cylindrical source, axis cutting OC at right angles.
7. Short uniform cylindrical source, axis coinciding with OC.
8. Uniform source in the form of a short prism of square section, axis coinciding with OC.

9 and 10. As (7) and (8), with correction for absorption in the source included.

OC is the axis (produced) of the ionization chamber, assumed cylindrical in form.

DISCUSSION.

Prof. O. W. **Richardson** complimented the author on a very careful piece of work, which would be of value in industrial measurements.

Dr. E. A. **Owen** said that investigators using radium will be grateful for the Paper, which works out fully the most interesting case—that in which the tube is at right angles to the radium bench—and also the effect of scattering. Radium is usually estimated in a comparatively small room, and near the walls. It would appear that by keeping at a distance from walls the effect of scattering can be reduced by about one-half, and it is very useful to know the exact figures.

Mr. A. F. **Dufton** (communicated) : Mr. Backhurst's experimental results indicate that, owing to the effect of scattered radiation, the formulæ propounded are perhaps more of academic than of practical interest. In practice, the comparison of two radium preparations can be so effected that the obliquity correction is obviated. For example, in comparison with either a tube 4 in. long or a plaque 3 in. in diameter, a concentrated preparation can be placed 1 in. from the axis. At a reasonable distance from the ionisation chamber this displacement compensates for the distribution of the radium and the application of a correction is avoided.

Author's reply (communicated) : From the experiments it will be seen that in order to be able to calculate the obliquity correction with a reasonable degree of accuracy without the necessity of any recourse to experiment, it is important to reduce the amount of scattered radiation to a minimum. Under the conditions of experiment obtaining in Fig. 3 for example, the error introduced by the effect of scattered radiation in the case of a tube 12 cm. long, placed centrally perpendicular to the axis, would amount to about ½ per cent., this being approximately one-third of the error for a point source at 6 cm. from the axis. Should it be necessary to allow for the effect of scattered radiation, the constant K_0 must be determined, for which, however, one experimental value is sufficient.

Mr. Dufton suggests that the obliquity correction may be obviated by displacing the standard radium source a suitable distance from the axis. One of the disadvantages of this method is that to obtain the same degree of accuracy afforded by reference to a table of corrections, the comparison experiments must occupy a considerably longer time unless the equivalent position of the standard source is correctly calculated for each case. It is clear that the terms involving T^4 in a formula such as No. 5 may often be neglected, and the correction taken as proportional to the square of the distance from the axis, but a linear relationship, such as he assumes, will not be nearly such a good approximation, as a glance at Fig. 3 will at once show. In the case he gives of a tube 4 in. long, the standard source being placed 1 in. from the axis, an error of about 0·4 per cent. would be made if the measurements were carried out at 25 cm. from an ionisation chamber of 14·5 cm. diameter and in the ideal case of no scattering. If scattering exists to a large extent as in the experiment illustrated in Fig. 4, the error made would be about 0·3 per cent. To reduce this error to about 0·1 per cent. the measurements could be carried out at 50 cm. distance, but the time for each observation would be four times as long. At 25 cm. distance an observation on a weak source seldom occupies less than 1 minute and frequently more than 2 minutes. From 60 to 120 observations are commonly taken on the two sources being compared. so that in order to obtain results to an equivalent degree of acuracy Mr. Dufton would have to spend from 3 to 12 hours longer by his method.

Two of the main objects of the Paper have been to show how a table may be compiled by reference to which such a loss of time may be avoided and how a moderate degree of scattering may be allowed for with fair accuracy. Mr. Dufton would appear to regard such considerations as mainly of " academic " importance.

XXIX.—VISCOSITY OF WATER AT LOW RATES OF FLOW DETERMINED COMPARATIVELY BY A METHOD OF THERMAL CONVECTION.

By ALBERT GRIFFITHS, *D.Sc.*, Professor of Physics, Birkbeck College, and P. C. VINCENT, *B.Sc.*, Research Student, Birkbeck College.

Received February 26, 1926.

ABSTRACT.

The Paper describes a method of determining the coefficient of viscosity of water by measuring the rate of convective flow in a long capillary tube, the driving head being obtained by the difference in density due to a difference of temperature between two vertical columns of water.

The results are in agreement with the conclusion reached in a previous Paper by Dr. Constance H. Griffiths and one of us—*viz.*, that the viscosity of water at low rates of shear in glass tubes is apparently no different from that at high rates.

The novel features of the Paper appear to be (1) the method of obtaining the driving head; (2) the new method of introducing a coloured index in a closed capillary circuit, and an improved method of reading its position; (3) the almost complete elimination of what may be called the thermometric effect as distinct from the convective effect; (4) the elimination of the small residual thermometric effect from the final calculations.

THIS Paper describes a method of determining the coefficient of viscosity of water at low rates of shear by measuring the flow due to thermal convection in a closed circuit. This circuit consists essentially of a modified rectangle, composed of two short wide vertical tubes and two long horizontal capillary tubes, and the flow is caused by a difference of temperature between the two vertical tubes. The method gives results consistent to a fraction of one per cent.

Each series of horizontal capillary tubes was, in the first place, standardized or calibrated by measuring the flows due to heads of water of the order of 60 cms.; and the rate of shear at the boundary of the capillary tubes was of the order of 120 radians per second. In standardizing these tubes Hosking's values* for the coefficient of viscosity of water were taken as correct.

The rate of shear at the boundary in the convective experiments was, in one experiment, as low as 0·012 radians per second; and the effective driving head for the upper and lower sets of tubes in series was of the order of 0·014 cms. of water.

The method is essentially comparative, and it involves the knowledge of the relation between the density and temperature of water.

A diagrammatic sketch of the apparatus is shown in Fig. 1. *AB* and *CD* are the vertical arms of the rectangle, and *E* and *F* are the long horizontal capillary arms. *E* and *F* are each about 19 metres long, and they are of a vertical distance of about 18 cms. apart.

AB and *CD* are placed in glass accumulator tanks *L* and *M* respectively. The tank *L* is heated electrically, and a convective current proceeds from *A* to *C* and from *D* to *B*.

X and *Y*, each of which comprises a pair of three-way taps, are index-introducers—i.e., they enable short columns of a feeble uranine solution to be introduced into the capillary tubes ; the linear movements with respect to time of these short coloured columns along the calibrated capillary tubes give the rate of flow of the convective current.

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* Phil. Mag., Vol. 18, p. 262 (1909).

The vertical tubes *AB* and *CD* have a considerable volume-capacity, and if the circuit were absolutely closed the changes in volume of the water due to variations in temperature might damage the apparatus and cause a leak ; but, quite apart from any damage to the apparatus, the changes in volume may cause a movement of the water, which is not usually called a convection-current and which may be called a " thermometric movement." This thermometric movement in *E* and *F* may be eliminated by the broad tubes *G* and *H*, which run from the wide vertical tubes into an open glass tank *K*, which is partially filled with water. These broad tubes are provided with taps *I* and *J*. If changes of temperature occur in *AB* only, say, then thermometric movements in the capillary tubes are prevented by opening *I* and closing *J*, for any expansion or contraction is satisfied by a movement into and out of *K*.

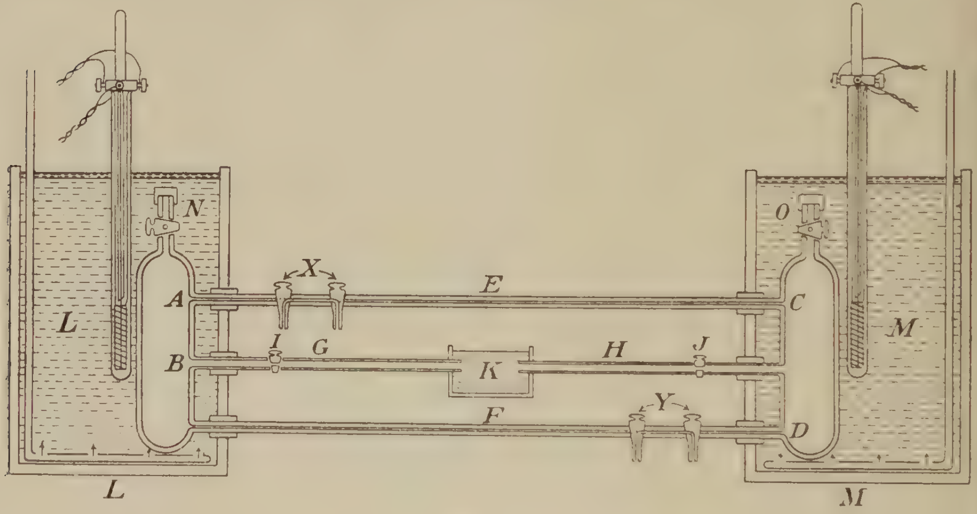

Fig. 1.

If the tap *J* is closed and changes of temperature occur in *CD* as well as in *AB*, then there are thermometric movements in the capillary tubes ; but by using two coloured indexes in the rectangular circuit, one travelling from *A* to *C*, the other from *D* to *B*, the effects due to the thermometric movement are eliminated from the final calculations ; for if an expansion, say, occurs in *CD* an increase in speed from *D* to *B* is balanced in the calculations by a decrease in speed from *A* to *C*.

Results may be, and have been, obtained with both the taps *I* and *J* open ; but with this arrangement the apparatus can hardly be said to be functioning as a simple closed circuit.

The water in the high-temperature tank *L* and the low-temperature tank *M* was kept thoroughly stirred by means of bubbles of air. A piece of compo tubing was so shaped that it fitted inside the tank parallel and close to one vertical edge, and parallel and close to three bottom horizontal edges. The further end of the tubing was sealed. Fine holes about 6 inches apart were bored along the length of the tubing at the bottom of the tank. The open end of the tubing was connected to a water pump which was so arranged that a gentle stream of air bubbles was forced through the water in the tanks.

Surface evaporation of the water in a tank was made very small by covering the surface with a layer of medicinal paraffin oil. Without this coating the changes of temperature caused by the evaporation due to the stirring was excessive and troublesome.

The vertical parts AB and CD have attached at their upper ends tubes N and O provided with stop-cocks. These tubes are for convenience in filling the apparatus. N and O are provided with indiarubber seals.

The temperature of the tank M was adjusted so as to be approximately that of the air of the room.

The capillary tubes were kept at a uniform temperature by an arrangement which cannot be explained with the aid of the diagrammatic sketch, for the actual shape of the apparatus was decidedly different from that shown in Fig. 1. The arrangement is explained later.

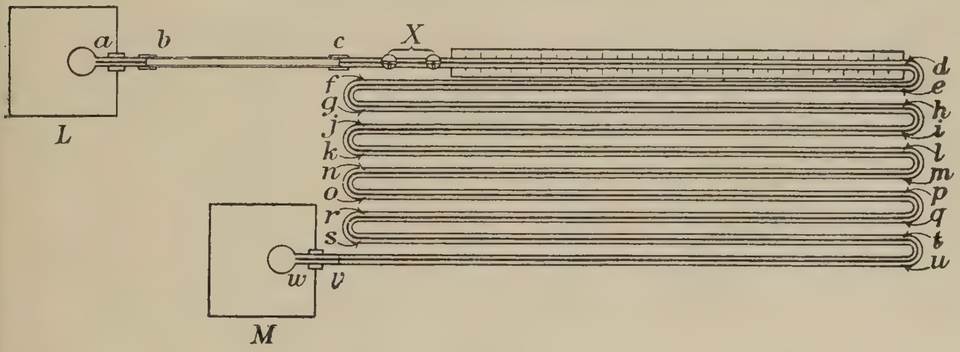

Fig. 2.

Fig. 2 shows, by means of a rough plan, the actual arrangement of the apparatus and the way the capillary tubes are connected to the tanks L and M.

The upper capillary tube is made of 12 lengths, cd, ef, . . . uv, each about 160 cms. long. These lengths are connected together by glass U-tubes with the aid of transparent indiarubber tubing, the ends of the tubes being ground flat, so as to make the fitting good. The series of tubes is supported on a stout wooden frame, the level of which can be adjusted by levelling screws. The tube cd is provided with an index introducer X. Under cd is fixed a black paper scale with division marks drawn with Chinese white. The scale is black to make the faint fluorescent light of the index easily visible. The tube cd is calibrated so far as volume-capacity is concerned by obtaining the mass of mercury occupying various lengths of the tube and dividing by the density of the mercury. Thus the volume in cubic centimetres between the zero and any division of the scale is known.

The high-temperature tank L radiates heat, and it is therefore advisable to place it at a considerable distance (50 cms. in many of the experiments recorded) from the series of capillary tubes. The connecting tube *bc* is of wide bore, and in consequence the viscous resistance to the flow of water along it is negligible ; and therefore the difference between its temperature and that of the capillary circuit may be neglected altogether, and it is not necessary to determine the variation of temperature along it—i.e., along the tube *bc*.

The tube *ab* is of capillary bore. It passes through an indiarubber stopper that is inserted in a hole, which has been sand-blasted through the walls of the tank. It is short ; and therefore no correction is required for its temperature.

The lower series of capillary tubes is arranged in an analogous manner to the upper one. It, in particular, has a tube analogous to *cd*, which has been carefully calibrated, and which contains an index-introducer.

The whole apparatus is in a thermostat-room ; but it was found in earlier experiments that the high-temperature tank *L* caused excessive variations of the temperature in the neighbourhood of the series of capillary tubes, especially in those parts of the tubes nearest the heater. In the experiments recorded a uniform temperature throughout the length of both series of tubes was obtained by means of an electric fan. This fan blew a stream of air at the general temperature of the thermostat-room towards the hot tank parallel to and through the system of tubes.

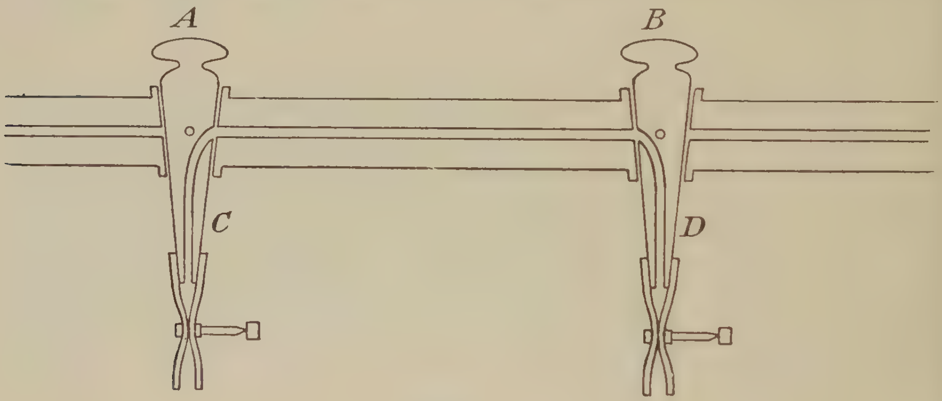

Fig. 3.

This uniformity of temperature is important, for a change of one degree centigrade causes the coefficient of viscosity of water, over the range of the experiments, to vary by more than 2 per cent.

It would probably be an improvement to have the capillary tubes in a large, well-stirred water-bath ; but this would involve many incidental changes in the apparatus, and would, moreover, be expensive.

Method of Introducing a Coloured Index into a Calibrated Tube.

A sketch indicating the method of introducing coloured indexes into the upper and lower series of capillary tubes is shown in Fig. 3.

Two three-way taps *A* and *B* are placed at a convenient distance apart (say 4 cms.). The openings *C* and *D* are provided with indiarubber tubes which can be sealed by means of clips.

To introduce an index a feeble solution of uranine is passed from C to D. The indiarubber tubes are now sealed with the clips and the tap-plugs are each turned through 90°.

The indiarubber seals to C and D are of vital importance, for without them there is generally sufficient leakage from one or both of the plugs to spoil an experiment.

In the case of the index-introducers used in the experiments described in this Paper, the taps A and B were at such a distance apart that the procedure described above could be adopted. If their distance apart should be too great a coloured index of convenient length may be obtained by allowing the flow to take place until a requisite length of coloured liquid has emerged from the space between the taps. The rear part of the coloured liquid may then be washed out by turning the tap-plugs A and B through 90°, undoing the screw clips, and sending a stream of water from C to D. Obviously, the tap-plugs A and B must now be turned back through 90°, and C and D again sealed.

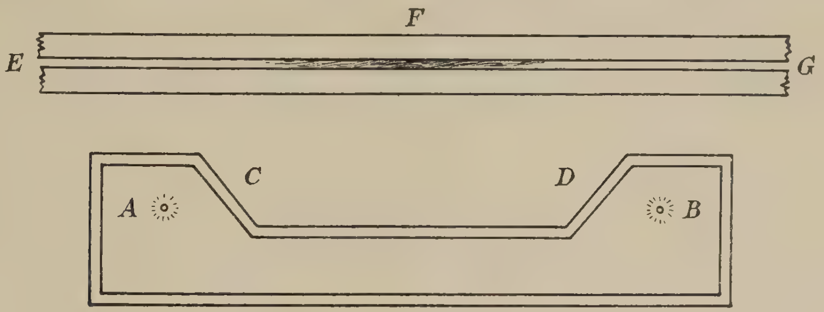

FIG. 4.

In the experiments recorded in this Paper the strength of the uranine solution was 0·5 grams to the litre.

METHOD OF READING THE COLOURED INDEX.

An improved method, involving the use of two incandescent lamps, was used to read the position of the coloured index.

In earlier unrecorded experiments the illumination consisted of a solitary lamp placed in a box which was provided with a window of cobalt glass. When the lamp was in a symmetrical position, as it should be, with regard to the coloured index, and the eyes of the observer were held over the fluorescent index, discomfort in reading was caused by the relatively powerful reflection from the outer wall of the capillary tube.

The source of illumination in the experiments recorded in this Paper is shown in Fig. 4.

Two lamps A and B were contained in a box provided with cobalt glass windows C and D. The capillary tube is indicated by EG, the fluorescent index by F. The illuminating apparatus is arranged in a symmetrical position with regard to the index, and it is clear that if the eyes of the observer are above F the rays from A and B through C and D respectively, which strike the outer wall of the capillary tube in the neighbourhood of the index, are reflected to right and left, and do not disturb the observer. The fluorescent light from the uranine is scattered in all directions, and can therefore be readily seen.

STANDARDIZATION OF EACH SERIES OF CAPILLARY TUBES AT A RATE OF SHEAR OF ABOUT 140 RADIANS PER SEC.

Let V_1=the volume-flow along the upper series (*AEC* in Fig. 1) in cubic centimetres per second.

V_2=the volume-flow along the lower series, *DFB*, in cubic centimetres per second.

P_1=the difference of pressure between the ends of *AEC* in dynes per square centimetre.

P_2=the difference of pressure between the ends of *DFB* in dynes per square centimetre.

φ=the temperature of the water in the capillary tubes.

$\mu\phi$=the coefficient of viscosity at $\varphi^{0}c$, obtained from Hosking's formula.*

For standardizing the tube use is made of constants K_1 and K_2 defined by

$$P_1=\mu\varphi K_1 V_1 \qquad \text{(1)}$$

$$P_2=\mu\varphi K_2 V_2 \qquad \text{(2)}$$

(it may be incidentally mentioned that $K=\frac{8l}{\pi a^4}$, where l is the length of a capillary tube and a the radius of the bore.)

The details of the apparatus used to standardize the capillary tubes are of no particular interest. The method is not an unusual one, and is sufficiently indicated by the figures given below.

Let t=time in seconds.

h_0=head of water in cms. when $t=0$.

h=head of water in cms. when $t=t$ secs.

ρ=density of water at temperature of experiment.

F=total volume-flow in time t in c.c.

Then it can be proved that

$$K=\frac{\rho g(h_0-h)t}{F\mu\varphi\ \log_e\frac{h_0}{h_1}}.$$

As an example, consider the upper series of tubes. In one experiment the details were as follows :—$h_0=69{\cdot}98$ cms., $h=65{\cdot}07$ cms., $t=2{,}580$ secs., $\rho=0{\cdot}999$ gm./cm.^{3}, $F=140{\cdot}457$, $\varphi=16^{0}{\cdot}08$, $\mu\varphi=0{\cdot}011124$.

The values of K_1 and K_2 only vary very slightly with the temperature—about 2 parts in 100,000 per degree. They may be taken as constants.

The average value of $K_1=1{\cdot}0942\times10^8$.

" " $K_2=1{\cdot}5320\times10^8$.

The average rate of shear at the boundary is given by $\frac{4F}{\pi a^3 t}$, and was of the order of 140 radians per second.

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* Phil. Mag., Vol. 18, p. 262 (1909).

THEORY OF THE FLOW DUE TO THERMAL CONVECTION.

Let T = the temperature of the high temperature tank L (*see* Fig. 5).
t = the temperature of the low-temperature tank M.
θ = the temperature of the capillary tubes.
H = the vertical distance in cms. between the centre of the capillary tubes near the high temperature tank L.
h = the vertical distance in cms. between the centres of the capillary tubes near the low temperature tank M.
v_1 = the volume-flow in cc. per sec. along AEC.
v_2 = the volume-flow in cc. per sec. along DFB.
ρ_T = the density of water at T°C.
ρ_t = density of water at t°C.
ρ_θ = density of water at θ°C.
η_θ = the coefficient of viscosity of water at θ°C.
p_a = the pressure at A in dynes per sq. cm.
p_b = ,, ,, B ,, ,,
p_c = ,, ,, C ,, ,,
p_d = ,, ,, D ,, ,,

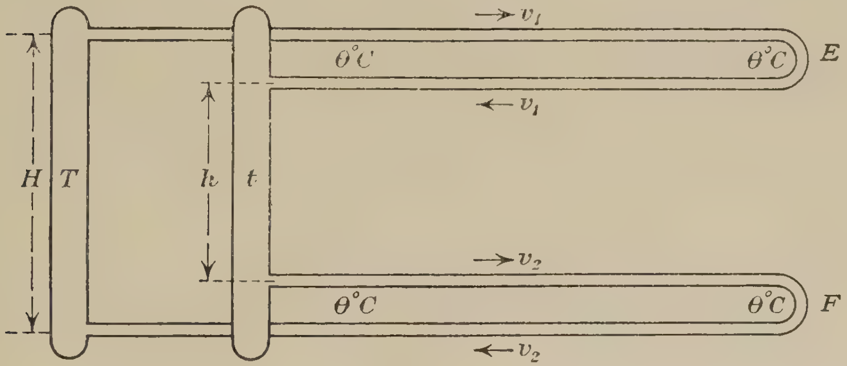

FIG. 5.

If the series of tubes E (Figs. 1 and 5) were absolutely horizontal, then $(p_a - p_c)$—i.e., the excess of the pressure at A above that at B would equal $\eta_\theta K_1 v_1$, as suggested by equation (1). If, however, the tubes are not horizontal, then, if A is at a height y_a above a given horizontal plane, not shown in the figures, the excess of the pressure is given by the equation

$$p_a - p_c = \eta_\theta K_1 v_1 + (y_c - y_a)\,\rho_\theta g \qquad (3)$$

Similarly, where p_d and p_b have obvious meanings

$$p_d - p_b = \eta_\theta K_2 v_2 + (y_b - y_d)\,\rho_\theta g \qquad (4)$$

Since the tubes AB and CD are very wide, the viscous resistance in them may be neglected, and hydrostatical considerations give

$$p_b - p_a = g\,\rho_T H \qquad (5)$$

$$p_d - p_c = g\,\rho_t h \qquad (6)$$

adding corresponding sides of (3) and (4), and making use of (5) and (6), we obtain

$$\eta_\theta = \frac{g\,\rho_t h - g\,\rho_T H + g\,\rho_\theta H - g\,\rho_\theta h}{K_1 v_1 + K_2 v_2}$$

By a little algebraic manipulation the equation may be put in the form

$$\eta_\theta = \frac{g(\rho_t - \rho_T)\,H + g(\rho_\theta - \rho_t)\,(H - h)}{K_1 v_1 + K_2 v_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This form of expression for η_θ is useful as indicating the effect of the horizontal tubes, etc., not being exactly horizontal.

If the tubes are exactly horizontal, $H=h$, and the second term in the numerator of equation (7) vanishes: for $(H-h)=0$.

If $H=|=h$, but the temperature of the tubes is the same as that of the low temperature tank, the second term again vanishes; for $\rho_\theta - \rho_t = 0$.

In practice in the experiments recorded in this Paper both $(\rho_t - \rho_T)$ and $(H-h)$ are small, and their product is so exceedingly small that the second term in the numerator may be neglected, and it is sufficiently exact to write

$$\eta_\theta = \frac{g(\rho_t - \rho_T)H}{K_1 v_1 + K_2 v_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It may be mentioned that generally v_1 does not differ greatly from v_2. If there is no thermometric motion v_1 should equal v_2.

In the course of a long experiment, when T, t, v_1 and v_2 vary, $(\rho_t - \rho_T)$ should be taken as the average value of the difference between the two densities, and v_1 and v_2 should be average rates of volume-flow per second.

With the small variations in temperature associated with the present experiments, to a sufficient degree of accuracy, (1) the average value of $(\rho_t - \rho_T)$, and (2) the value of ρ at the average value of t minus the value of ρ at the average value of T are equal.

Some of the calculations involved are suggested by the following Table I, which gives the figures for the series of observations denoted Experiment 1.

TABLE I.

θ	T	ρ_T	t	ρ	$(\rho_t - \rho_T)$	Time
17·67°C.	43·14°C.	0·991012	17·66°C.	0·998684	0·00767153	12½
17·63	43·18	0·990998	17·69	0·998678	0·00768120	12¾
17·59	43·21	0·990983	17·72	0·998673	0·00768970	13
17·57	43·30	0·990946	17·69	0·998677	0·00773135	13¼
17·55	43·40	0·990908	17·67	0·998681	0·00777322	13½
17·55	43·46	0·990883	17·80	0·998658	0·00777508	13¾
17·62	43·54	0·990850	17·81	0·998656	0·00780598	14
17·67	43·66	0·990801	17·85	0·998649	0·00784796	14¼
17·75	43·68	0·990793	17·88	0·998644	0·00785096	14½
17·75	43·73	0·990771	17·90	0·998639	0·00786819	14¾
17·78	43·77	0·990756	17·93	0·998634	0·00787803	15
17·84	43·80	0·990744	17·94	0·998633	0·00788944	15¼
17·86	43·82	0·990735	18·00	0·998622	0·00788715	15½
17·82	43·84	0·990725	18·01	0·998621	0·00789567	15¾
17·85	43·86	0·990717	18·04	0·998614	0·00789746	16
17·85	43·88	0·990711	18·07	0·998609	0·00789859	16¼
17·85	43·91	0·990698	18·09	0·998605	0·00790638	16½
17·84	43·92	0·990694	18·13	0·998597	0·00790300	16¾
17·85	43·94	0·990684	18·15	0·998593	0·00790932	17
17·83	43·98	0·990667	18·17	0·998590	0·00792270	17¼
17·86	43·97	0·990674	18·21	0·998583	0·00790936	17½
17·92	43·96	0·990676	18·25	0·998578	0·00790267	17¾
17·85	43·97	0·990672	18·25	0·998574	0·00790224	18
17·90	43·94	0·990684	18·28	0·998569	0·00788509	18¼
17·90	43·94	0·990684	18·28	0·998569	0·00788509	18½
17·77	43·71	0·990779	17·98	0·998625	0·00784665	Averag

(1) Average value of $(\rho_t - \rho_T) = 0{\cdot}00784665$.

(2) Value of ρ at average value of t minus the value of ρ at average value of $T = 0{\cdot}00784749$.

The figures for the densities are interpolated from tables given in the 1905 edition of Landolt-Bornstein Physikalisch-Chemische Tabellen.

It may be noted that the error involved in taking the average value of $(\rho_t - \rho_T)$ as the difference between the densities at the average temperatures is of the order of 0·01 per cent. Thus the arithmetical calculations may be abbreviated enormously. They could be still further abbreviated for approximate work by first calculating the average platinum thermometer temperatures and transforming these to the hydrogen-thermometer scale.

DETAILS OF THE CONVECTIVE EXPERIMENTS.

The distilled water in the apparatus was in each case boiled in a copper vessel, then put under an exhaust pump, and finally a trace of copper sulphate was added to the water. This treatment was to sterilise the water and to preserve it from the development of fungoid growths.

The temperatures of the tanks *L* and *M* were read every quarter of an hour by means of platinum thermometers, when the convective flow lasted five or six hours, and were read every two hours (day and night) when the experiment lasted days. The platinum thermometer temperatures were reduced to the gas-thermometer scale in the usual way.

The temperature of the air in the neighbourhood of the capillary tubes was read about the same time as the tank temperatures by means of mercury thermometers. The same thermometers were used during the standardization of the capillary tubes at rapid flow, and, as the method is a comparative one, any small absolute error or correction in these thermometers is of little importance.

The Griffiths-Callendar bridge which was used in conjunction with the platinum thermometers, and the telescopes which were used for reading the mercury thermometers, were placed behind a screen; and thus the thermal effect of the observers, which had caused considerable difficulty in preliminary experiments, was made negligible.

EXPERIMENT 1.—SEPTEMBER 22, 1925.

The apparatus was cleaned before the experiment, the duration of which was about six hours. The apparatus was not quite as indicated in the diagram, the long tube *bc* being missing; but the distance from the capillary systems to the heating tank *L* was sufficiently great.

EXPERIMENT 2.—NOVEMBER 5, 1925.

New water was introduced in the convective apparatus and the experiment lasted about five hours. The arrangement was the same as in Experiment 1.

EXPERIMENT 3.—NOVEMBER 6, 1925.

The same water was used as in Experiment 2, and the conditions were the same.

EXPERIMENT 4.—DECEMBER 7, 1925.

The apparatus was cleaned and new water was added. The arrangement was like that described in the text, with the addition of a cork mat placed so as to protect the low temperature tank from the radiation emitted by the high temperature tank. The experiment lasted five hours.

EXPERIMENT 5.—DECEMBER 18-20, 1925.

The same water was used as in Experiment 4, and the conditions were the same, except that there was a much smaller difference of temperature between the two tanks, and thus a much reduced rate of flow.

It may be noted that the average excess is of the order of magnitude of minus one-tenth per cent.

The values of the viscosity at low rates of shear (see Table II) differ from those at high rates of shear by less than would be expected from the errors to which the various measurements are liable; and there is no suggestion that the viscosity of water at low rates of shear in glass tubes is different from that at high rates of shear.

TABLE II.—*Summary of Experimental Results.*

Date	Sept. 22.	Nov. 5.	Nov. 6.	Dec. 17.	Dec. 18-20.
Time in secs. ...	21,229	18,960	19,140	18,060	200,872
Rate of flow, v_1 ...	$4{\cdot}914 \times 10^{-5}$	$4{\cdot}708 \times 10^{-5}$	$6{\cdot}163 \times 10^{-5}$	$4{\cdot}266 \times 10^{-5}$	$4{\cdot}852 \times 10^{-6}$
Rate of flow, v_2 ...	$4{\cdot}904 \times 10^{-5}$	$4{\cdot}585 \times 10^{-5}$	$6{\cdot}089 \times 10^{-5}$	$4{\cdot}353 \times 10^{-5}$	$4{\cdot}951 \times 10^{-6}$
Average value of θ...	17·77°C.	17·38	17·61	17·93	16·77
K_1	$1{\cdot}0942 \times 10^{8}$	$1{\cdot}0942 \times 10^{8}$	$1{\cdot}0942 \times 10^{8}$	$1{\cdot}0942 \times 10^{8}$	$1{\cdot}0942 \times 10^{8}$
K_2	$1{\cdot}5320 \times 10^{8}$	$1{\cdot}5320 \times 10^{8}$	$1{\cdot}5320 \times 10^{8}$	$1{\cdot}5320 \times 10^{8}$	$1{\cdot}5320 \times 10^{8}$
Average value of T	43·71°C.	42·346°C.	48·292°C.	41·087°C.	20·972°C.
Average value of t ...	17·98°C.	17·377°C.	17·781°C.	17·645°C.	16·799°C.
Average value of $(\rho_t - \rho_T)$	$7{\cdot}847 \times 10^{-3}$	$7{\cdot}328 \times 10^{-3}$	$9{\cdot}817 \times 10^{-3}$	$6{\cdot}861 \times 10^{-3}$	$8{\cdot}108 \times 10^{-4}$
H in cms.	17·78	17·78	17·78	17·80	17·80
Approx. rate of shear in radians per sec.	0·13	0·13	0·16	0·12	0·013
Hosking's values of coefficient of viscosity	0·01062	0·01074	0·01068	0·01058	0·01090
Experimental values of coefficient of viscosity	0·01062	0·01067	0·01065	0·01056	0·01095
Excess above Hosking's	0·0 p.c.	−0·7 p.c.	−0·3 p.c.	−0·2 p.c.	+0·5 p.c.

This conclusion is the same as that arrived at by a different method,* in which the effective driving head was mainly due to the difference between the density of a weak uranine solution and that of pure water.

To the authors the chief points of the present work appear to be :—

(1) The proof that a considerable degree of control has been obtained in the production of a small calculable pressure-head of different values in a closed circuit containing water, by means of a difference of temperature between two vertical columns of water.

(2) The absence of any suggestion (in accordance with numerous unpublished observations with a preliminary form of the apparatus) that the viscosity of water at low rates of shear is different from that at high rates of shear.

(3) The arrangement for minimizing the thermometric movement in the circuit.

(4) The method of using two coloured indexes in appropriate parts of the circuit, thus eliminating the effects due to the slight thermometric movement from the final calculation.

(5) The new device by introducing a coloured index in a closed circuit, and the special type of lamp for reading the position of the coloured index.

In conclusion, the authors wish gratefully to acknowledge the assistance of Messrs. H. G. Bell, H. Broan, F. Staley and G. Steeden, and to express their appreciation of the suggestions and help of Dr. Constance H. Griffiths.

DISCUSSION.

Prof. O. W. RICHARDSON said that the authors had used an ingenious method of getting a controlled flow of water at an extremely slow rate, which might have many useful applications in research.

Prof. A. GRIFFITHS said that in an earlier investigation, made in conjunction with Dr. Constance Griffiths, irregularities had been found which must be due either to temperature errors or to what had been called in the present Paper the "thermometric effect." This Paper helped to throw light on that subject, but the work described formed part of rather a long series of determinations, and he therefore felt that the Society had exhibited a certain amount of good nature in receiving it.

Mr. F. E. SMITH said that, on the contrary, many Fellows of the Society had a great admiration for the extremely accurate and thorough work which was being carried out under Prof. Griffiths direction. Apparently nothing else of the same kind was being done in this country, and it was only by extensive experimentation that all the properties of liquids could be recorded.

Mr. L. F. RICHARDSON : Sandström has stated that it is impossible for heat to produce a steady circulation unless the source of heat is at a level lower than that of the sink of heat. At first sight these experiments seem to contradict this statement, because the hot and cold vertical tubes are on the same level. But on consideration of the readings in Table II it is clear that the cooling occurred mainly near where the water entered the upper capillary, while the heating occurred mainly at the lower end of the warm vertical column. Thus there is no conflict with Sandström's statement.

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* "Viscosity of Water at Low Rates of Shear," by Albert Griffiths and Constance H. Griffiths, Proc. Phys. Soc., Vol. 33, Part 4, June 15 (1921).

XXX.—THE PROPERTIES OF MUTUAL INDUCTANCE STANDARDS AT TELEPHONIC FREQUENCIES.

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Abstract.

It is well known that when a mutual inductance carries alternating current, the value of the effective mutual inductance varies with the frequency, and further that the secondary P.D. is not exactly in quadrature with the primary current. Thus certain frequency corrections are introduced into all alternating-current bridges in which a mutual inductance standard is used. An experimental investigation has been made of the actual magnitude of such corrections for mutual inductance standards of three types :—

(1) A stranded wire fixed standard, with spaced winding to minimise capacity effects.
(2) The Campbell variable mutual inductance standard.
(3) The Tinsley variable mutual inductance standard made in accordance with Butterworth's recommendations.

Values of the corrections are given for all these instruments under various conditions. It is shown that whereas the variation of mutual inductance with frequency is mainly due to capacity effects, dielectric losses in the insulation, or alternating current conductance, play an important part in the determination of the phase defect, the impurity being found to be proportional to a power of the frequency higher than the second, over the telephonic range of frequency.

Introductory.

MUTUAL inductance standards may be used in alternating-current bridge work for making a great variety of measurements. They may be used for measuring capacity or inductance, in which case they may be regarded as standards of reactance. They may also be used in potentiometer methods for measuring phase angles, e.g., the power factor of a condenser, or the phase angle of a standard shunt. In such cases they may be regarded as standards of phase angle.

In using a mutual inductance as a standard of phase angle we make use of the fact that when alternating current passes through the primary coil, the E.M.F. induced in the secondary coil is exactly in quadrature with the current in the primary coil. The quantity we are concerned with in bridge and potentiometer measurements is not so much the E.M.F. induced in the secondary coil, as the potential difference between its terminals. In a perfect mutual inductance the P.D. between the terminals of the secondary coil on open circuit would be equal and opposite to the E.M.F. induced in that coil, but in practice, owing to the existence of capacity and eddy-current effects in the secondary coil, and also capacity effects between the primary and secondary coils, the P.D. between the secondary-coil terminals is not exactly equal and opposite to the induced E.M.F., but differs somewhat from this both in magnitude and phase. Further, as mentioned above, the E.M.F. induced in the secondary coil by a definite current flowing through the primary coil is exactly in quadrature with that current, but owing to distributed capacity and eddy-current effects in the primary coil and between the two coils, the current is not exactly the same in every element of the primary coil. The current we measure in practice is the current flowing into one primary terminal and out of the other. If earth-capacity effects are not suitably distributed these two currents will not be exactly equal, but here earth-capacity effects will not be considered, since they are not so much a property of the instrument itself as of its position and potential distribution. Accordingly, we shall consider the primary current I_p as flowing into one primary terminal and out of the other. It is obvious from what has been said that the P.D. (V_s) between the secondary terminals on open circuit will not usually be in quadrature with I_p. We may write

$$V_s=(\sigma+jM\omega)I_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where M is the effective mutual inductance, σ is the impurity, $\omega/2\pi$=frequency, and j is the operator rotating through a right angle. We may regard $\theta=\tan^{-1}\frac{M\omega}{\sigma}$ as being the phase angle of such a mutual inductance. It is very nearly equal to 90 deg. The small angle $\delta=90 \text{ deg.}-\theta=\tan^{-1}\frac{\sigma}{M\omega}$ may be regarded as the phase defect of the instrument. This Paper gives an account of an experimental investigation into the properties of actual mutual inductance standards of various types with special reference to the impurity and change of effective mutual inductance at the higher telephonic frequencies.

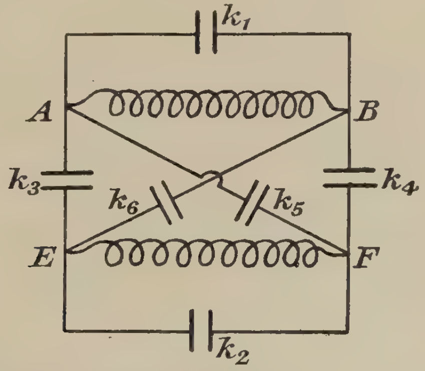

FIG. 1.—EQUIVALENT NETWORK FOR MUTUAL INDUCTANCE STANDARD.

GENERAL THEORETICAL CONSIDERATIONS.

Butterworth* has investigated mathematically the effect of eddy currents and capacity effects on the effective mutual inductance and impurity of two coils. Let AB, EF (Fig. 1) be the two coils. As an approximation the distributed-capacity effects in the system are represented by the capacities k_1, k_2, k_3, k_4, k_5, k_6, which are regarded as concentrated at the ends of the coils. In order to simplify the problem further, suppose the two coils have a common point, say, AE. The capacity k_3 is now short circuited and disappears, k_1 and k_6 are thrown in parallel and may be represented by a single capacity C_1, k_2 and k_5 similarly form the single capacity C_2, and we have the system of Fig. 2, in which $C_{12}=k_4$. Butterworth shows that this system possesses the following properties.

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* Butterworth, Proc. Phys. Soc., Vol. 33, p. 312 (1921).

Let L_1R_1 be the resistance and self-inductance of the primary coil, L_2R_2 those of the secondary coil, and M_0 the true or geometrical mutual inductance between them, then the effective mutual inductance and impurity as defined by Equation (1) are given by

$$M=M_0+C_{12}R_1R_2+\omega^2\{C_1L_1M_0+C_2L_2M_0-C_{12}(L_1-M_0)(L_2-M_0)\} \quad . \; . \; . \quad (2)$$

$$\sigma=\omega^2[C_1R_1M_0+C_2R_2M_0-C_{12}\{R_1(L_2-M_0)+R_2(L_1-M_0)\}] \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

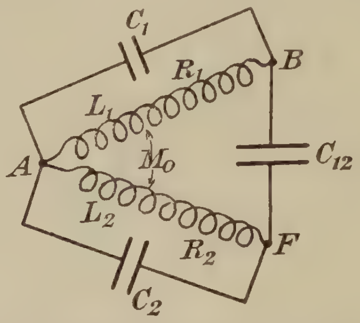

FIG. 2.—SIMPLIFIED NETWORK WHEN THE TWO COILS HAVE ONE POINT COMMON.

These expressions only hold when the capacity effects are comparatively small, i.e., when terms such as $C_1\omega$ are small compared with those such as $\frac{1}{L_1\omega}$, small quantities of the second order being negligible. Further the sign of M_0 may be either positive or negative. If the direction of winding of the two coils is such that when current flows in at one open end, through the coils, and out of the other, the mutual inductance opposes the self-inductance, M_0 must be taken as positive. If the mutual inductance assists the self-inductance under these conditions, M_0 must be taken as negative. We shall accordingly speak of positive and negative mutual

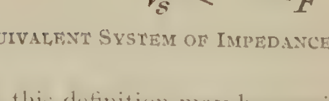

FIG. 3.—EQUIVALENT SYSTEM OF IMPEDANCE OPERATORS.

inductances. In order that this definition may be consistent with equation (1), we must suppose that the secondary potential difference V_s is reckoned in a definite direction, viz., when I_p flows towards the common point, V_s is also directed towards the common point (*see* Fig. 3), i.e., in the diagram F is considered to be at a higher potential than A. If the secondary coil is contained in a closed circuit, so that the induced E.M.F. produces a current, this current will flow from F to A in the part of the circuit external to the coil, and from A to F through the coil, i.e., the E.M.F. induced in the coil must be regarded as acting in the direction AF, as shown by the dotted arrow.

Equation (2) gives the change of effective mutual inductance with frequency as a result of capacity effects. At very low frequencies the effective mutual inductance is equal to the geometrical mutual inductance M_0, since the term involving ω^2 is then negligible, while the term $C_{12}R_1R_2$ is nearly always negligible. As the frequency increases, M increases by an amount

$$\triangle M=\omega^2\{C_1L_1M_0+C_2L_2M_0-C_{12}(L_1-M_0)(L_2-M_0)\} \quad . \; . \; . \; . \; . \quad (4)$$

For convenience $\frac{\triangle M}{M_0}$ is often referred to as the frequency coefficient of the mutual inductance. Thus

$$\text{Frequency coefficient}=\frac{\triangle M}{M_0}=\omega^2\left\{C_1L_1+C_2L_2-C_{12}\,.\,\frac{(L_1-M_0)(L_2-M_0)}{M_0}\right\} \quad . \; . \quad (5)$$

We may regard $L_1C_1\omega^2$ as the frequency coefficient due to the self-capacity of the primary coil, and $L_2C_2\omega^2$ that due to the self-capacity of the secondary coil. Both these terms are always positive. The frequency coefficient due to the mutual capacity between the two coils is $-C_{12}\frac{(L_1-M_0)(L_2-M_0)\,\omega^2}{M_0}$, and this term may be either positive or negative, depending on the relative value of L_1, L_2 and M_0. When M_0 is negative, this term is always positive, and is numerically equal to $C_{12}\frac{(L_1+M_0)(L_2+M_0)}{M_0}$. When M_0 is positive, and $L_1>M_0$, $L_2>M_0$, which is commonly the case in practice, the term is negative, and thus the frequency coefficient due to mutual capacity opposes that due to self-capacity. If $L_1>M_0>L_2$, the term is positive, but is numerically much smaller than the case in which M_0 is negative, provided C_{12} is the same in the two cases. Thus a positive mutual inductance has a much smaller frequency coefficient than a negative one, the capacities being the same in the two cases. This is an important practical point, since a mutual inductance standard can readily be changed from positive to negative by changing the common point, and in some bridges M may be of either sign. In such cases it is nearly always preferable to use a positive M.

Similar considerations apply to the phase defect due to capacity

$$\delta_c=\tan^{-1}\frac{\sigma_c}{M\omega}=\frac{\sigma_c}{M\omega}\text{ (approx.).}$$

Equation (3) gives

$$\delta_c=R_1C_1\omega+R_2C_2\omega-C_{12}\left\{R_1\frac{(L_2-M_0)}{M_0}\omega+R_2\frac{(L_1-M_0)\,\omega}{M_0}\right\} \quad . \; . \; . \quad (6)$$

and from this it follows that $\frac{\sigma_c}{M\omega}$ is always positive for a negative mutual inductance. In the case of a positive mutual inductance, the phase defect due to mutual capacity opposes that due to self-capacity, when $L_2>M_0$ and $L_1>M_0$, and in all cases the impurity and phase defect of a positive mutual inductance are less than those in a negative mutual inductance, if the capacities are the same in the two cases. When we are dealing with one particular pair of coils, the sign of M may be changed by simply changing the common point; but by reference to Figs. 1 and 2 we see that this change also alters the values of C_1, C_2 and C_{12}, unless the whole arrangement is perfectly symmetrical, which is not probable. Thus, when the sign of M is changed, the frequency coefficient and impurity are liable to change merely on account of the alteration in the values of C_1, C_2 and C_{12}. Nevertheless, in almost all the cases met with in practice, the negative connection possesses much the larger frequency coefficient and impurity.

Butterworth* has also shown that the effect of eddy currents in the coils is to introduce another component of impurity σ_e. This may be positive or negative, depending on the method of winding of the coils; but, in general, standards of mutual inductance are wound with stranded wire, and the effects of eddy currents are of less importance than those due to capacity.

METHODS OF MEASUREMENT.

In setting out to measure the properties of a number of typical standards of mutual inductance, it is important to have a method of measurement of effective mutual inductance and impurity at various frequencies, which is rapid in operation and which can be used for a wide range of frequencies. The methods used have been described in a previous Paper.† Two alternating-current potentiometer methods are there described. By means of one method two mutual inductances of the same sign, and not necessarily of the same magnitude, can be compared for impurity and effective mutual inductance at any desired frequency. The other method, the simplest possible, permits the comparison of two mutual inductances, equal in magnitude but opposite in sign. In both cases the impurity of one mutual inductance is given directly in ohms, provided the impurity of the other is negligibly small, or at any rate known. The quantities which it was desired to investigate were the frequency coefficient, or change of effective mutual inductance with frequency, and the impurity. The procedure was as follows: The instrument under test was connected in a circuit by means of which it could be balanced against a mutual inductance standard, which was known to possess a very small impurity and frequency coefficient. Balance was first obtained at a low frequency—say 100 cycles per second—using a vibration galvanometer as detector. At this low frequency the impurity of any ordinary mutual inductor is sensibly zero, and its effective mutual inductance is practically equal to its geometrical value M_0. The frequency was then raised to, say, 1,000 cycles, and a telephone was substituted for the vibration galvanometer as detector. The change to the higher frequency causes a change in the effective mutual inductance, and also introduces an impurity, and the arrangement is no longer balanced. The instruments tested were all variable mutual inductors, and thus balance at the higher frequency was obtained by reducing the mutual inductance reading by an amount $\triangle M$, and at the same time altering the impurity adjustment r. If the impurity and frequency coefficient of the standard are negligible, then $\triangle M$ gives at once the change of mutual inductance with frequency of the instrument under test, whilst the impurity is obtained at once from the change in the value of r. The frequency was then again raised, and the values of $\triangle M$ and impurity found for the higher frequency, and so on. In this way the frequency coefficient and impurity of the instrument were readily obtained for the whole range of telephonic frequencies.

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* Butterworth, loc. cit.

† Journal of Scientific Instruments, Vol. 2, p. 145 (1925).

In order to work the method successfully, it is necessary to have a high frequency standard of mutual inductance—that is to say, a standard whose properties at high frequency are as nearly as possible the same as at low frequencies. For this purpose two coils such as are used in radio frequency work were frequently used. These coils were wound of highly stranded wire, so that eddy current effects were very small, and they were "slice" wound, so that capacity effects were reduced to a minimum. Two such coils of the same diameter and placed side by side possess a mutual inductance which is very nearly pure and invariable with frequency over the telephonic range. The value of the mutual inductance is conveniently varied by moving one coil away from the other along their common axis. They must not, of course, be turned so that a portion of the winding of one coil comes into the strongest part of the field of the other, otherwise eddy current effects will be increased. For testing 10mH mutual inductometers, a fixed standard mutual inductance designed by Mr. D. W. Dye was used. The coils are wound on an ebonite frame of cylindrical form. This is "slice" wound with stranded wire (size 7/36's), the whole forming a coil of self inductance about 50·5mH. The winding is broken at two points, thus dividing it into three sections (Fig. 4). The middle section serves as the secondary coil, while the other two coils connected in series form the primary. Thus the secondary is sandwiched in between the two halves of the primary, thereby securing a comparatively high coupling coefficient, and thus enabling the 10mH of mutual inductance to be obtained with correspondingly less wire.

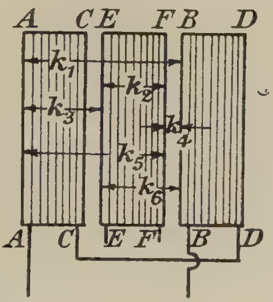

Fig. 4.—Arrangement of Coils in High Frequency Mutual Inductance Standard.

Determination of Residual Effects in the Standard.

For many purposes this standard may be regarded as a perfectly pure mutual inductance of constant value. There will, however, be certain small capacity effects even at telephonic frequencies. In particular, the mutual capacities are likely to be rather high on account of the closeness of coupling, and these mutual capacities affect the values of C_1, C_2 and C_{12}, as previously explained. Eddy current effects also may not be absolutely negligible. An idea of the magnitude of these residual effects was obtained in the following manner.

(*a*) *Self-capacity Measurements at Radio Frequencies.*

Let one primary terminal be connected to one of the secondary terminals, so that the system of Fig. 2 is obtained. Butterworth* has shown that this is equivalent to the system of Fig. 5, where X, Y and Z are vector impedances (assuming we are dealing with alternating currents of frequency $\omega/2\pi$), such that the following relations are satisfied,

$$X,\ Y,\ Z=\frac{x+\alpha\mu,\ y+\beta\mu,\ z+\gamma\mu}{1+\alpha(y+z)+\beta(z+x)+\gamma(x+y)+\lambda\mu}$$

where

$$x=j\omega M_0=m,\quad y=R_1+jL_1\omega-jM_0\omega=l-m,\quad z=R_2+jL_2\omega-jM_0\omega=n-m,$$
$$\alpha=j\omega C_{12}\qquad \beta=j\omega C_2\qquad \gamma=j\omega C_1,$$
$$\mu=yz+zx+xy=ln-m^2\qquad \lambda=\beta\gamma+\gamma\alpha+\alpha\beta.$$

Let now a variable air condenser be connected across the primary coil (i.e., across AB in Fig. 2), and let the coil be loosely coupled to an oscillating circuit.

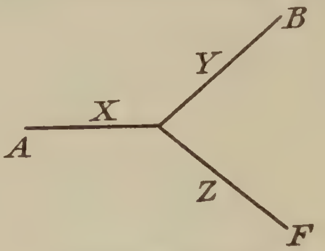

FIG. 5.—EQUIVALENT STAR CONNECTION.

Let the capacity of the added condenser be varied until the circuit resonates. If this is done for a number of frequencies, the relation between the added capacity and the corresponding resonating frequency can be obtained. The circuit will resonate at the frequency $\omega/2\pi$, such that the reactance of the oscillatory circuit formed is zero. This reactance consists of the reactance component of $X+Y$, together wtith that of the added condenser. From the above equations we obtain for the impedance $X+Y$,

$$X+Y=\frac{l+(\alpha+\beta)(ln-m^2)}{1+\alpha(l+n-2m)+\beta n+\gamma l+\lambda\mu}$$

The capacities C_1, C_2, C_{12} are small, and if the frequency is well below the self-resonance points of the various coils, the terms α, β, γ are small compared with $\frac{1}{l}$, $\frac{1}{m}$, $\frac{1}{n}$, i.e., terms such as $l\alpha$, $n\beta$, $l\gamma$, etc., are small compared with unity. If we neglect squares and products of such small quantities the above expression reduces to

$$X+Y=l-\alpha(l-m)^2-\beta m^2-\gamma l^2$$
$$=(R_1+jL_1\omega)-j\omega C_{12}[R_1+j(L_1-M_0)\omega]^2+j\omega C_2M_0^2\omega^2-j\omega C_{12}[R_1+jL_1\omega]^2.$$

The reactance component of this expression is

$$\omega[L_1-(C_1+C_{12})R_1^2+\omega^2\{C_1L_1^2+C_2M_0^2+C_{12}(L_1-M)^2\}].$$

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* Butterworth, loc. cit.

The second term in this expression will be negligibly small for all reasonably good coils, and thus it can be omitted. The reactance of the added condenser is $-1/C\omega$. Thus, the condition for resonance is to a first approximation

$$\omega L_1\left[1+\omega^2\left\{C_1L_1+C_2\frac{M_0^2}{L_1}+C_{12}\frac{(L_1-M_0)^2}{L_1}\right\}\right]-\frac{1}{C\omega}=0,$$

or,

$$\frac{1}{\omega^2}=CL_1\left[1+\omega^2\left\{C_1L_1+C_2\frac{M_0^2}{L_1}+C_{12}\frac{(L_1-M_0)^2}{L_1}\right\}\right]$$

$$=L_1\left[C+\left\{C_1+C_2\frac{M_0^2}{L_1^2}+C_{12}\frac{(L_1-M_0)^2}{L_1^2}\right\}\right]\text{(approx.)},$$

since, in the second term, which is small, we may put $L_1C\omega^2=1$ with sufficient approximation. Thus, on plotting corresponding values of C and $1/\omega^2$ we get a straight line, which cuts the axis of C at the point $C=-C_p$, such that

$$C_p=C_1+C_2\frac{M_0^2}{L_1^2}+C_{12}\frac{(L_1-M_0)^2}{L_1^2} \quad \dots\dots\dots\dots \tag{7}$$

and which is inclined to the C axis at an angle $\tan^{-1}L_1$. Evidently we may regard C_p as the effective self-capacity of the primary coil in the presence of the secondary, and may determine it in the manner indicated. Similarly the effective self-capacity of the secondary in the presence of the primary is given by

$$C_s=C_2+C_1\frac{M_0^2}{L_2^2}+C_{12}\frac{(L_2-M_0)^2}{M_0^2} \quad \dots\dots\dots\dots \tag{8}$$

and this may be determined in the same way. Finally, by connecting a variable condenser across BF (Fig. 2), and observing resonance points as before, the effective self-capacity of the primary and secondary in series (C_{ps}) may be determined. In this case the impedance of the coils between the terminals BF is given by

$$Y+Z=\frac{(l+n-2m)+(ln-m^2)(\beta+\gamma)}{1+\alpha(l+n-2m)+\beta n+\gamma l+\lambda\mu}$$

Neglecting small quantities of the second order, this becomes

$$Y+Z=l_s+(\beta+\gamma)(ln-m^2)-\alpha l_s^2-\beta l_s n-\gamma l_s l,$$

where

$$l_s=y+z=l+n-2m=R_1+R_2+j\omega(L_1+L_2-2M_0).$$

The reactance component of the above impedance is

$$\omega[L_1+L_2-2M_0-\{C_1R_1^2+C_2R_2^2+C_{12}(R_1+R_2)^2\}+\omega^2\{C_1(L_1-M_0)^2 \\ +C_2(L_2-M_0)^2+C_{12}(L_1+L_2-2M_0)^2\}].$$

Again, the term involving the resistance is negligibly small. Let C be the added capacity, and denote $L_1+L_2-2M_0$ by L_s, then the condition of resonance is

$$\frac{1}{\omega_1}=CL_s\left[1+\omega^2\left\{C_1\frac{(L_1-M_0)^2}{L_s}+C_2\frac{(L_2-M_0)^2}{L_s}+C_{12}L_s\right\}\right]$$

Thus, the self-capacity C_{ps} is given approximately by

$$C_{ps}=C_{12}+C_1\frac{(L_1-M_0)^2}{L_s^2}+C_2\frac{(L_2-M_0)^2}{L_s^2} \quad . \; . \; , \; . \; . \; . \; . \; . \qquad (9)$$

Thus, if L_1, L_2, and M_0 are known, and we measure C_p, C_s, and C_{ps}, in this way, we have three equations (7), (8) and (9), connecting C_1, C_{12}, and C_2, from which we can obtain their values.

Measurements were made in this way on the standard mutual inductance. Fig. 4 represents the geometry of this, while Fig. 1 represents its electrical constants. It is evident from Fig. 1 that four common points are possible, and by making the appropriate connections we obtain four different mutual inductances, two positive and two negative (the geometrical value is, of course, the same in each case).

The values observed for the various effective self-capacities are given in Table I. Substituting the values C_p, C_s, C_{ps} for any given common point in equations (7), (8), (9), and solving, the corresponding values of C_1, C_2, C_{12} were obtained. These are also given in Table I. It is interesting to note that the measured effective self-capacities are much greater when M_0 is negative than when it is positive. This is due to the large change in the numerical value of the terms (L_1-M_0), etc., in equations (7), (8) and (9). The component capacities C_1, C_2, C_{12}, are of the same order in the two cases. Immediately underneath each component capacity in Table I is given the combination of the various end-capacities in Fig. 1, which go to form that particular value of C_1, C_2 or C_{12}. From this it is at once evident that k_4, the capacity between B and F is the greatest of these end-capacities. The reason for this is apparent on inspection of Fig. 4; the points BF are the nearest together. The capacity k_5 between the points A and F is very much smaller, corresponding to their greater distance apart. It is also evident from the results in Table I that we cannot regard the capacities k_1, k_2, k_3, etc., as strictly additive when the connections are such as to throw them in parallel. The capacity C_1 is certainly largest when AF is the common point, in which case the large capacity k_4 is thrown across the primary,

TABLE I.

Sign of M.	Com. Pt.	Effective Self-capacities.			Component Capacities.		
		C_p	C_s	C_{ps}	C_1	C_2	C_{12}
		micro-microfarads.			micro-microfarads.		
+	*AE*	33	22	70	19 k_1+k_6	6 k_2+k_5	57 k_4
+	*BF*	16	32	32	5 k_1+k_5	25 k_2+k_6	30 k_3
−	*AF*	130	170	55	60 k_1+k_4	26 k_2+k_3	30 k_6
−	*BE*	57	100	26	21 k_1+k_3	60 k_2+k_4	8 k_5

$L_1=18{\cdot}5$ mH. $L_2=12{\cdot}0$ mH. $M_0=\pm10{\cdot}0$ mH.

but in general, where a capacity may be expected to be equal to the sum of two end-capacities, such as k_1+k_6, the value actually observed is smaller than the sum would be, judging by the values given for the single capacities k_3, k_4, k_5, k_6 as obtained from the values of C_1, C_2, C_{12}. This is partly explained by the fact that the capacities k_1, k_2 represent actions which take place across the same dielectric path ; if they were to be strictly additive, each would have to be completely self-contained and shielded from the others.

Using these values of the various capacities, we can now calculate the values of σ and $\frac{\Delta M}{M_0}$, using equations (3) and (5). The values are given in Table II.

TABLE II.—*Frequency Coefficient and Impurity of High Frequency Fixed Mutual Inductance Standard.* (*Values calculated from capacities.*)

Sign of M.	Com. Pt.	Frequency.	$\frac{\Delta M}{M}$	σ (ohm)	$\frac{\sigma}{M\omega}$
+	*AE*	1,000 $\sim$	$0{\cdot}1\times10^{-4}$	$-0{\cdot}0001$	$-0{\cdot}2\times10^{-5}$
		2,000 $\sim$	$0{\cdot}5\times10^{-4}$	$-0{\cdot}0005$	$-0{\cdot}4\times10^{5}$
		4,000 $\sim$	$2{\cdot}0\times10^{-4}$	$-0{\cdot}002$	$-0{\cdot}8\times10^{-5}$
+	*BF*	1,000	$0{\cdot}1\times10^{-4}$	$0{\cdot}000_0$	...
		2,000	$0{\cdot}6\times10^{-4}$	$0{\cdot}000_0$	...
		4,000	$2{\cdot}2\times10^{-4}$	$-0{\cdot}000_1$	-4×10^{-7}
—	*AF*	1,000	$1{\cdot}3\times10^{-4}$	$-0{\cdot}001_3$	$+2{\cdot}_1\times10^{-5}$
		2,000	$5{\cdot}_2\times10^{-4}$	$-0{\cdot}005$	$+4\times10^{-5}$
		4,000	$21{\cdot}_0\times10^{-4}$	$-0{\cdot}021$	$+8\times10^{-5}$
—	*BE*	1,000	$0{\cdot}_5\times10^{-4}$	$-0{\cdot}000_5$	$+0{\cdot}_8\times10^{-5}$
		2,000	$2{\cdot}_1\times10^{-4}$	$-0{\cdot}002$	$+1{\cdot}_6\times10^{-5}$
		4,000	$8{\cdot}_3\times10^{-4}$	$-0{\cdot}008$	$+3{\cdot}_2\times10^{-5}$

A check on the accuracy of these results was obtained in the following way. The standard was balanced against a variable mutual inductance of the same value using the simple comparison method, and readings of M and r noted, at a telephonic frequency. The connections to the primary and secondary coils of the standard were then each reversed, thus changing the common point, but not changing the sign of M. Balance could therefore again be obtained by adjusting the variable mutual inductance and also the impurity adjustment, just sufficiently to compensate for the differences in the impurity, and frequency change of the standard, for the two conditions of common points, i.e., the method is one of simple substitution since the frequency errors of the variable mutual inductance are sensibly the same in the two measurements. The results of such measurements are given in Table III, where $\left(\frac{\Delta M}{M_0}\right)_{AF}-\left(\frac{\Delta M}{M_0}\right)_{BE}$ represents the difference of the frequency corrections of the standard when the common points are AF and BE. Within the experimental error, the calculated and observed results are identical. In Table III, the values σ_{AF} and σ_{BE} must be considered as representing the impurity due to capacity effects.

only, since even in the case of the observed values, although eddy-current effects may also be appreciable, they may be expected to be the same in the two cases, and thus will cancel out on taking differences.

TABLE III.

Frequency.	$\left(\frac{\Delta M}{M}\right)_{AF}-\left(\frac{\Delta M}{M}\right)_{BE}$		$\sigma_{AF}-\sigma_{BE}$	
	Calculated.	Observed.	Calculated.	Observed.
1,000	$0{\cdot}_8\times10^{-4}$	$0{\cdot}_4\times10^{-4}$	$-0{\cdot}001$	$-0{\cdot}001$
2,000	$3{\cdot}_1\times10^{-4}$	$2{\cdot}_6\times10^{-4}$	$-0{\cdot}003$	$-0{\cdot}004$
4,000	$10{\cdot}_8\times10^{-4}$	$11{\cdot}_8\times10^{-4}$	$-0{\cdot}01_1$	$-0{\cdot}0_2$

The importance of eddy-current effects was estimated by the following experiments :—

(*b*) *Effective Resistance Measurements at Radio Frequencies.*

This method was used by Butterworth. The effective resistances of the coils at radio frequencies were measured by the ordinary resistance substitution method. As before, let the coils have a common point. Let R_1 be the effective resistance of the primary coil AB (Fig. 3) at any given frequency, let R_2 be the effective resistance of the secondary AF at the same frequency, then the resistance of the two coils in series (BAF) at the same frequency is $R_1+R_2-2\sigma$, where σ is the impurity of the mutual inductance between the coils at that frequency. Thus the total σ can be measured at a number of high frequencies, and the value at telephonic frequency obtained by extrapolation. Values obtained for the standard in this way are given in Table IV for two conditions of common point. Under the heading "impurity" column (*a*) gives the results obtained directly from the effective resistance measurements at the frequencies stated, column (*b*) gives the values obtained for various telephonic frequencies by extrapolation from the results in column (*a*). In column (*c*) the corresponding values of σ_c are given for comparison. These values are taken from Table II. The values in column (*a*) were found to vary as a higher power of the frequency than the square. This is probably due to dielectric losses in the insulation and will be referred to again later. The relation $\sigma=\sigma_0\omega^{2{\cdot}45}$ was found to hold approximately, and this was used in extrapolating. Even if the square law is assumed, however, the general conclusions are not affected, viz., that the component of σ due to eddy currents is less than 0·01 ohm at 4,000 ∿, and is therefore negligible for all practical purposes at telephonic frequencies. This follows at once from a comparison of the results in columns (*b*) and (*c*), and may be expected with the stranded wire used. Thus the values given in Table II may be taken as completely representing the behaviour of the standard at telephonic frequencies. When used as a positive mutual inductance, the impurity is negligible, while the value of the mutual inductance increases by 2 parts in 10,000 in passing from zero frequency (or, say, 100 ∿) to 4,000 ∿. When used as a negative mutual inductance, the impurity just becomes appreciable, whilst in going from 100 ∿ to 4,000 ∿, the mutual inductance increases by 2 parts in 1,000 in one case. The importance of knowing this is obvious, since in practice when testing a positive mutual inductance by the simple comparison method (the most satisfactory), the standard must be used as a negative mutual inductance.

TABLE IV.

Sign of M.	Com. Pt.	Frequency.	Impurity σ—ohms.		
			Observed. (*a*)	By extrapolation. (*b*)	From capacity measurements. (*c*)
+	*AE*	37,400	0·4		
		30,000	+0·2		
		20,000	$+0{\cdot}0_5$		
		4,000		$+0{\cdot}00_1$	−0·002
		2,000		$+0{\cdot}000_2$	$-0{\cdot}000_5$
		1,000		$+0{\cdot}000_0$	$-0{\cdot}000_1$
−	*AF*	25,000	−1·9		
		20,000	−1·0		
		10,000	−0·2		
		4,000		$-0{\cdot}02_1$	$-0{\cdot}02_1$
		2,000		−0·004	−0·005
		1,000		$-0{\cdot}000_8$	$-0{\cdot}001_3$

WORKING STANDARDS OF MUTUAL INDUCTANCE.

The Campbell inductometer, or variable mutual inductance standard, is very widely used for alternating-current bridge work, and accordingly it appeared to be advisable to determine its properties in some detail. Butterworth has given the frequency corrections for a 10mH instrument, at the top point of its scale, at 1,000 cycles; but, in addition to this, it is important to know how the frequency corrections vary with scale reading, change in common point, and also with the frequency itself. These points were investigated by comparison with the standard mutual inductance described above. This standard consisted of three coils (Fig. 4), and by using various combinations of these coils values of about 2mH, 5mH, 7mH and 10mH could be obtained. These were compared directly with the Campbell instrument set at the same reading. For the purposes of these experiments, it was usually sufficiently accurate to assume that the standard mutual inductance was perfect. Tables V and VI give the results obtained with the 10mH Campbell instrument. Table V gives the results for a common point which makes the mutual inductance positive, while Table VI gives values for a common point corresponding to a negative value for M. The following points are to be noted :—

(1) The increase of mutual inductance ($\triangle M$) with increase of frequency is always proportional to the square of the frequency within the experimental error.

(2) When the mutual inductance is positive, the fractional increase of mutual inductance with frequency is approximately constant (at any given frequency) over the whole range of the instrument. This does not hold when the mutual inductance is negative.

(3) The impurity σ in nearly every case increases more rapidly than the square of the frequency. Butterworth showed that the impurity due to capacity and eddy-current effects is proportional to the square of the frequency

under the conditions of these experiments. It is evident that there must be an additional component of impurity, which is proportional to a power of the frequency higher than the square. It is shown in a succeeding paragraph that this may be explained by dielectric losses in the insulating material in the coils.

(4) When the mutual inductance is positive the phase defect of this instrument is approximately constant at any given frequency for all readings of mutual inductance. This is very convenient in practice. For instance, in determining the power factor of a condenser by means of the Carey Foster bridge, using the instrument, the correction to be applied for the impurity of the mutual inductance will always be approximately the same for a given frequency. It is merely necessary to add the phase defect of the instrument to the observed value of the power factor.

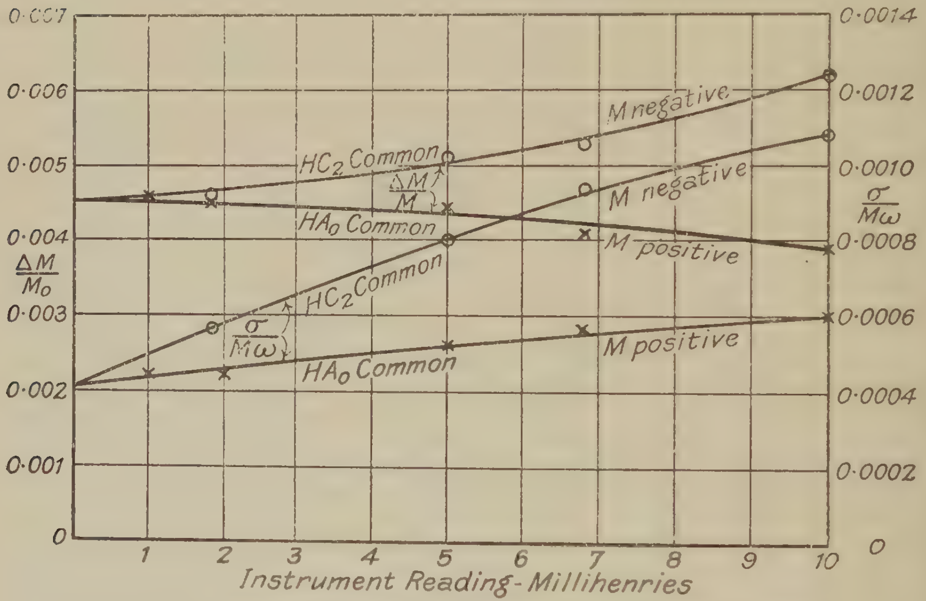

FIG. 6.—FREQUENCY CORRECTIONS OF 10MH CAMPBELL INDUCTOMETER.
Frequency = 2000 ~.

(5) When the reading of the instrument is near the top of its scale, the frequency corrections for the negative connection are much larger than for the positive; but as the instrument reading becomes smaller the correction for the two methods of connection become more and more nearly equal. Fig. 6 shows this very clearly. There the frequency corrections at a frequency of 2,000 cycles are plotted against instrument reading for both methods of connection. The differences between the corrections for large values of M are evidently due to the effect of mutual capacity, as previously explained.

Tables V and VI gives the results for two common points. Two other common

TABLE V.—*Frequency Corrections of* 10mH *Campbell Inductometer.*
Common point HA_0. Mutual inductance positive.

Instrument reading M_0.	Frequency.	ΔM	σ (ohms).	$\frac{\Delta M}{M_0}$	$\frac{\sigma}{M\omega}$
1,000 μH	500	$0\cdot_3\mu H$	$+0\cdot000_4$	$2\cdot_9\times10^{-4}$	$0\cdot0001_1$
	1,000	$1\cdot_2$	$+0\cdot001_4$	$12\cdot_6\times10^{-4}$	$0\cdot0002_2$
	1,500	$2\cdot_7$	$+0\cdot003_2$	$26\ \times10^{-4}$	$0\cdot0003_8$
	2,000	$4\cdot_8$	$+0\cdot005_7$	$46\ \times10^{-4}$	$0\cdot0004_4$
1,820	500	$0\cdot_5$	$+0\cdot000_7$	$2\cdot_8\times10^{-4}$	$0\cdot0001_2$
	1,000	$2\cdot_0$	$+0\cdot002_5$	$11\ \times10^{-4}$	$0\cdot0002_2$
	1,500	$4\cdot_5$	$+0\cdot006$	$25\ \times10^{-4}$	$0\cdot0003_5$
	2,000	$8\cdot_1$	$+0\cdot010$	$45\ \times10^{-4}$	$0\cdot0004_4$
5,000	500	$1\cdot_3$	$+0\cdot002_6$	$2\cdot_6\times10^{-}$	$0\cdot0001_3$
	1,000	$5\cdot_4$	$+0\cdot007_0$	$11\ \times10^{-4}$	$0\cdot0002_2$
	1,500	$12\cdot_1$	$+0\cdot017$	$24\ \times10^{-4}$	$0\cdot0003_6$
	2,000	$21\cdot_7$	$+0\cdot033$	$44\ \times10^{-4}$	$0\cdot0005_2$
6,820	500	$1\cdot_7$	$+0\cdot002_8$	$2\cdot_5\times10^{-4}$	$0\cdot0001_3$
	1,000	7	$+0\cdot011$	$10\ \times10^{-4}$	$0\cdot0002_5$
	1,500	15	$+0\cdot026$	$22\ \times10^{-4}$	$0\cdot0004_1$
	2,000	28	$+0\cdot048$	$41\ \times10^{-4}$	$0\cdot0005_6$
10,000	500	$2\cdot_5$	$+0\cdot004$	$2\cdot_5\times10^{-4}$	$0\cdot0001_3$
	1,000	$9\cdot_8$	$+0\cdot016$	$9\cdot_8\times10^{-4}$	$0\cdot0002_6$
	1,500	22	$+0\cdot040$	$22\ \times10^{-4}$	$0\cdot0004_3$
	2,000	39	$+0\cdot075$	$39\ \times10^{-4}$	$0\cdot0006_0$

TABLE VI.—*Frequency Corrections* 10mH *Campbell Inductometer* (*continued*).
Common point HC_2. Mutual inductance negative.

Instrument reading μH.	Frequency.	ΔM (μH)	σ (ohms).	$\frac{\Delta M}{M_0}$	$\frac{\sigma}{M\omega}$
1,820	500	$0\cdot_5$	$-0\cdot000_7$	$2\cdot_5\times10^{-4}$	$0\cdot0001_2$
	1,000	$2\cdot_0$	$-\ \cdot002_6$	$11\ \times10^{-4}$	$0\cdot0002_3$
	1,500	$4\cdot_4$	$-0\cdot006_7$	$24\ \times10^{-4}$	$0\cdot0003_9$
	2,000	$8\cdot_4$	$-0\cdot013$	$46\ \times10^{-4}$	$0\cdot0005_7$
5,000	500	$1\cdot_2$	$-0\cdot002_3$	$2\cdot_4\times10^{-4}$	$0\cdot0001_4$
	1,000	$5\cdot_8$	$-0\cdot010$	$12\ \times10^{-4}$	$0\cdot0003_2$
	1,500	$13\cdot_0$	$-0\cdot026$	$26\ \times10^{-4}$	$0\cdot0005_5$
	2,000	$25\cdot_4$	$-0\cdot050$	$51\ \times10^{-4}$	$0\cdot0008_0$
6,820	500	2	$-0\cdot003_9$	$2\cdot_9\times10^{-4}$	$0\cdot0001_8$
	1,000	9	$-0\cdot017$	$13\ \times10^{-4}$	$0\cdot0004_0$
	1,500	20	$-0\cdot041$	$29\ \times10^{-4}$	$0\cdot0006_3$
	2,000	36	$-0\cdot080$	$53\ \times10^{-4}$	$0\cdot0009_3$
10,000	500	$3\cdot_8$	$-0\cdot006_0$	$3\cdot_8\times10^{-4}$	$0\cdot0001_9$
	1,000	15	$-0\cdot028$	$15\ \times10^{-4}$	$0\cdot0004_5$
	1,500	35	$-0\cdot067$	$35\ \times10^{-4}$	$0\cdot0007_0$
	2,000	62	$-0\cdot134$	$62\ \times10^{-4}$	$0\cdot0010_7$

points are possible. The corrections were determined in each case for the reading M_0=10mH. The results are collected in Table VII.

TABLE VII.—*Frequency Corrections of Campbell Inductometer for Various Common Points.*
Instrument reading: 10mH. Frequency: 1,000∼.

Common point.	Sign of M.	$\frac{\Delta M}{M_0}$	σ	$\frac{\sigma}{M\omega}$
HA_0	+	$9{\cdot}_8\times10^{-4}$	$+0{\cdot}016$	$0{\cdot}0002_6$
FC_2	+	$8{\cdot}_2\times10^{-4}$	$+0{\cdot}015$	$0{\cdot}0002_4$
HC_2	−	$15{\cdot}_8\times10^{-4}$	$-0{\cdot}028$	$0{\cdot}0004_5$
FA_0	−	$11{\cdot}_8\times10^{-4}$	$-0{\cdot}021$	$0{\cdot}0003_3$

It is evident that it is important always to use a common point the corrections for which are known.

The values in Tables V, VI and VII all refer to the 10mH Campbell instrument, which is the most generally useful size of inductometer. The corrections for a 1mH instrument are somewhat smaller, since the capacity effects are naturally of less importance in coils of smaller self-inductance. Values for a 1mH instrument at the top of its scale are given in Table VIII. Here, again, the phase angle error is by no means negligible at 1,000 cycles.

TABLE VIII.—*Frequency Corrections of* 1mH *Mutual Inductance (Campbell type).*
Frequency: 1,000∼.

Common point.	Sign of M.	$\frac{\Delta M}{M_0}$	σ	$\frac{\sigma}{M\omega}$
HA_0	+	$1{\cdot}_9\times10^{-4}$	$+0{\cdot}001_5$	$0{\cdot}0002_4$
EC	+	$1{\cdot}_7\times10^{-4}$	$+0{\cdot}001_5$	$0{\cdot}0002_4$
EA_0	−	$2{\cdot}_4\times10^{-4}$	$-0{\cdot}001_7$	$0{\cdot}0002_7$
HC	−	$2{\cdot}_5\times10^{-4}$	$-0{\cdot}001_7$	$0{\cdot}0002_7$

In the case of the 100mH inductometer of the Campbell type the frequency corrections become much more serious. At 1,000∼ the correction on mutual inductance amounts to 1 per cent., and the phase defect is about 0·001. In fact, this instrument is not at all suitable for high-frequency work, and its properties have not therefore been examined in such detail. Actual values obtained for a 100mH Campbell inductometer are given in Table IX. Values of $\frac{\Delta M}{M_0}$ are given for a common point, making M positive, and also for one making it negative. When the instrument was used with the middle point of the primary connected to one point on the secondary (a convenient connexion for a self-inductance bridge), the frequency correction was found to be intermediate between those obtained for M positive and M negative respectively. The frequency coefficient $\frac{\Delta M}{M_0}$ was not approximately constant for different M_0 readings in the case of this instrument. For example, at the reading M_0=100mH, $\frac{\Delta M}{M_0}$ was about 1 per cent. at 1,000∼. At the reading M_0=50mH it amounted to 0·5 per cent., while for all readings below 10mH it was 0·3 per cent. At a given setting, M increased slightly more rapidly than the square of the frequency, but σ increased much more rapidly than the square of the frequency, as is evident from the values given in Table IX.

TABLE IX.—*Frequency Corrections of* 100mH *Campbell Inductometer.*

Common point.	Sign of M.	Frequency.	ΔM mH	σ (ohms).	$\frac{\Delta M.}{M_0}$	$\frac{\sigma}{M\omega}$
DE	+	500	0·24	$0{\cdot}08_2$	$0{\cdot}2_4\times10^{-2}$	$0{\cdot}0002_6$
		1,000	1·03	0·60	$1{\cdot}0_3\times10^{-2}$	$0{\cdot}0009_5$
		1,500	2·41	1·99	$2{\cdot}4_1\times10^{-2}$	0·0021
		2,000	4·79	4·6	$4{\cdot}7_9\times10^{-2}$	0·0035
DF	—	1,000	1·42	...	$1{\cdot}4_2\times10^{-2}$	...

EFFECT OF DIELECTRIC LOSSES ON THE FREQUENCY CORRECTIONS.

Consider the general case of a mutual inductor with a common point. Let $\xi\eta$ represent the resistance operators of the coils apart from capacity effects and the dielectric losses associated with them. Let their mutual operator be m. The capacity, dielectric loss, and leakage effects may be represented by shunts across the coils, whose conductance operators are α and β respectively, and the corresponding mutual effects (leakage and capacity) may be represented by a conductance (operator γ) across the open ends of the coils. Butterworth has shown that this system may be reduced by a series of transformations to the system of Fig. 3 in which we have two simple arms (resistance operators ξ' and η') with a mutual operator m', but no shunting conductances. When the terms, α, β, γ are small compared with ξ, η, m, which is the case in all mutual inductance standards at telephonic frequencies, the equivalent operators are given by the approximate equations

$$\left.\begin{aligned} m'&=m+\alpha(\xi-m)(\eta-m)-\beta m\eta-\gamma m\xi \\ \xi'&=\xi-\alpha(\xi-m)^2-\beta m^2-\gamma\xi^2 \\ \eta'&=\eta-\alpha(\eta-m)^2-\beta\eta^2-\gamma m^2 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (10)$$

Considering the case of alternating currents of frequency $\frac{\omega}{2\pi}$ we may replace these operators by vector impedances or conductances. Let R_1, R_2 be the resistances of the coils L_1, L_2 M the inductances. The conductance terms α, β, γ must include self-capacity, leakage, and dielectric loss terms. Thus, we may write

$$\left.\begin{matrix} \alpha=G_1+j\omega C_1 & \beta=G_2+j\omega C_2 & \gamma=G_3+j\omega C_3 \\ \xi=R_1+j\omega L_1 & \eta=R_2+j\omega L_2 & m=j\omega M \end{matrix}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (11)$$

We may also write

$$\left.\begin{aligned} m'&=\sigma+j\omega(M+\triangle M) \\ \xi'&=R_1+\triangle R_1+j\omega(L_1+\triangle L_1) \\ \eta'&=R_2+\triangle R_2+j\omega(L_2+\triangle L_2) \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

where $\triangle M$, σ, $\triangle L$, etc., may be regarded as the effects of self-capacity, leakage, etc., on resistance, inductance, or impurity. Substituting (11) in (10), and then comparing the result with (12), we find

$$\begin{aligned} \triangle M=C_{12}R_1R_2+\omega^2\{L_1C_1M+L_2C_2M-C_{12}(L_1-M)(L_2-M)\}-R_1G_1M-R_2G_2M \\ +G_{12}\{R_2(L_1-M)+R_1(L_2-M)\} \quad . \; . \; . \quad (13) \end{aligned}$$

$$\sigma=\omega^2[C_1R_1M+C_2R_2M-C_{12}\{R_1(L_2-M)+R_2(L_1-M)\}]+G_1L_1M\omega^2+G_2L_2M\omega^2$$
$$-\omega^2G_{12}(L_1-M)(L_2-M)+G_{12}R_1R_2\,. \quad . \quad . \quad (14)$$

$$\triangle L_1=-(C_1+C_{12})R_1{}^2+\omega^2\{C_1L_1{}^2+C_2M^2+C_{12}(L_1-M)^2\}-2G_1L_1R_1$$
$$-2G_{12}R_1(L_1-M) \quad . \quad (15)$$

$$\triangle R_1=2\omega^2R_1\{C_1L_1+C_{12}(L_1-M)\}-(G_1+G_{12})R_1{}^2+\omega^2\{G_1L_1{}^2+G_2M^2$$
$$+G_{12}(L_1-M)^2\}. \quad . \quad . \quad . \quad (16)$$

Thus, the frequency coefficient $\frac{\triangle M}{M}$ involves capacity terms of the form $L_1C_1\omega^2$ and also conductance terms of the form G_1R_1. G_1 represents not only leakage such as would be measured by a direct current test, but also the dielectric losses. In a well-constructed coil the D.C. leakage is very small compared with the alternating-current conductance due to dielectric losses. If δ is the loss angle of the insulation then $\tan\delta_1=\frac{G_1}{C_1\omega}$. G_1 is usually small compared with $C_1\omega$ (e.g., $\tan\delta$ is not likely to exceed 0·05 in a good coil), and R_1 is usually small compared with $L_1\omega$. Thus terms such as G_1R_1 are small compared with those such as $L_1C_1\omega^2$, and, therefore $\triangle M$ will depend almost entirely on the capacity terms. Further, since C_1C_2, etc., are practically independent of the frequency, $\triangle M$ will be proportional to the square of the frequency. This fact is verified by the experimental results quoted above.

In the expression for σ the capacity terms are of the form $C_1R_1M\omega^2$, whilst the conductance terms are of the form $G_1L_1M\omega^2$. When the inductance L_1 is high, this term is by no means negligible compared with the capacity term. Also, since G_1 is not even approximately independent of the frequency, this term will not be proportional to the square of the frequency. In a well-constructed coil, the conductances G_1, G_2, etc., will be almost entirely due to dielectric absorption effects in the insulated covering of the wire forming the coils. The conductance due to such effects varies with frequency, at any rate over the telephonic range, in accordance with the formula*

$$G_1=G_0{}'\omega^a,$$

where $G_0{}'$ is independent of frequency, and a is a constant, for a given material under definite conditions as to temperature, etc. In general, $0<a<1$. Thus, the conductance terms in σ are of the form $G'_0L_1M\omega^{2+a}$, and consequently σ increases more rapidly than the square of the frequency. This was found to be the case even for a 10mH inductometer, showing that dielectric loss effects were not negligible even in that case. In the case of the 100mH inductometer, it was evident that the observed σ was mainly due to conductance arising from dielectric losses since σ is approximately proportional to $\omega^{2.9}$. It is evident that the increase in effective resistance of the primary coil, $\triangle R_1$ will be similarly affected by conductance, while the effect on the change in effective inductance of the primary coil, $\triangle L_1$, will, like that on $\triangle M$, be very small.

THE BUTTERWORTH-TINSLEY INDUCTOMETER.

The sub-division of the dials in the Campbell type of inductometer is usually obtained by winding one coil with a cable of ten strands twisted together, putting the ten strands in series to form the reading 10 of the dial, and taking tappings from the junctions of the various strands to the studs 1, 2, 3, . . . etc. By this arrangement only one adjustment is necessary for each dial, and the accuracy of sub-division is of a very high order. This method of winding does, however, introduce large capacity effects into the coil in question, and the frequency errors are correspondingly large at high frequencies. Butterworth suggested that for work at high frequencies, a more perfect mutual inductance might be obtained by using for each dial, three mutual inductances with a common primary, the values being the 6, 3 and 1 of the dial in question. These three coils must be independently adjusted, but the other points on the dial are obtained by addition and subtraction of the three, e.g., 10 is given by 6+3+1, 9 by 6+3, 8 by 6+3 −1, and so on. It is, of course, necessary to devise a form of switch for connecting up the coils in the proper manner. The coils not required at any particular setting of the dial, are left isolated—e.g., at the setting 9, the 6 and 3 coils are connected in series, and the 1 coil is disconnected.

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* See, for example, McLeod, Phys. Rev., Vol. XXI., p. 53 (1923) ; Granier, Bull. de la Soc. Francaise des Electriciens, Vol. III., p. 333 (1923).

The capacity effects in coils of this construction are much smaller than in the stranded cable coils, and thus the frequency corrections are correspondingly smaller. An inductometer constructed in this manner by the firm of Messrs. Tinsley & Co. was tested in the same way as the Campbell instrument previously referred to. The values obtained are given in Table X. The frequency corrections are seen to be of the order of one-third to one-seventh of the corresponding corrections in the Campbell instrument, and the new instrument has, therefore, considerable advantages for high-frequency work. The Campbell instrument has, however, important advantages for standardising work which can be carried out at low frequency. It was, of course, designed for such work since the accuracy of sub-division of the dials is usually greater, and the form of dial used readily lends itself to the calibration of each reading in terms of the maximum reading, without the aid of another standard.

TABLE X.—*Frequency Corrections of* 10 *mH Butterworth-Tinsley Inductometer.*

Common Point.	M reading.	Frequency. ∼ /sec.	ΔM (μH).	σ (ohm).	$\frac{\Delta M}{M}$	$\frac{\sigma}{M\omega}$
D.C.	$+5{,}000\,\mu H$	1,000	$0{\cdot}7$	$0{\cdot}001_5$	$1{\cdot}_3\times10^{-4}$	$0{\cdot}_5\times10^{-4}$
		2,000	$2{\cdot}_6$	$0{\cdot}007$	$5{\cdot}_2\times10^{-4}$	$1{\cdot}_1\times10^{-4}$
	$+10{,}000\,\mu H$	1,000	$1{\cdot}_3$	$0{\cdot}004$	$1{\cdot}_3\times10^{-4}$	$0{\cdot}_6\times10^{-4}$
			$5{\cdot}_2$	$0{\cdot}015$	$5{\cdot}_2\times10^{-4}$	$1{\cdot}_2\times10^{-4}$
D.B.	$-10{,}000\,\mu H$	1,000	$-3{\cdot}_1$	$-0{\cdot}008_7$	$3{\cdot}_1\times10^{-4}$	$1{\cdot}_3\times10^{-4}$
		2,000	-12	$-0{\cdot}004_4$	12×10^{-4}	$3{\cdot}_5\times10^{-4}$
		4,000	-51	$-0{\cdot}21$	51×10^{-4}	$8{\cdot}_4\times10^{-4}$

It is evident that dielectric losses are also appreciable in this instrument, since σ (when *DB* is common) increases more rapidly than the square of the frequency.

In conclusion, I wish to express my thanks to Messrs. D. W. Martin and D. A. Oliver, of the Electrical Department, N.P.L., for assistance in taking some of the observations recorded in this Paper.

DISCUSSION.

Mr. A. CAMPBELL (communicated): I warmly congratulate Mr. Hartshorn on his success in solving the difficult problem of the measurement of small impurity in mutual inductances. He has skilfully got round the difficulty by working downwards from radio frequencies, which exaggerate the effects in question. Of the two methods of comparison used by the author, the one for equal inductances was introduced by me many years ago for the purpose of balancing the out-of-phase component, before "impurity of M" had been exactly defined, and it has proved useful even for extremely impure mutual inductances, such as iron-cored current transformers. Some time after my publication of the method (N.P.L. Report for 1908), Professor A. Larsen, of Copenhagen, extended it, and made it the basis of his beautifully simple alternating-current potentiometer. It is interesting to notice that, if the detecting instrument and the current source are interchanged it becomes Mr. Dye's method of testing voltage transformers.

The comparison of unequal mutual inductances is a good deal more difficult; Mr. Hartshorn's new method for this is excellent, and, being a great advance on the older methods, it should be regarded as the standard procedure. The author's results will afford the users of strand-subdivided inductometers suitable corrections where these are required; it should be remembered, however, that it is only in rather extreme cases (such as for the power losses in very perfect condensers or in very highly inductive coils) that the errors are of much consequence. As a means of obtaining quick and accurate subdivision of mutual inductances the stranding system still holds its own, having been for many years the key to the easy dissemination of very accurately subdivided standards. Of late years I have constructed inductometers by other more laborious systems, and I trust that when one of the new type passes through the author's hands it will be found fairly free from impurity errors.

AUTHOR'S reply: I am grateful to Mr. Campbell for his kind appreciation of my Paper. I was unaware of the exact history of the method for comparing equal mutual inductances, although I knew that Mr. Campbell's Papers treat the properties of the "MR element" very thoroughly and that he had used it in a variety of measurements some years ago. I shall look forward with interest to the appearance of his new inductometer.

XXXI.—A NOTE ON λ4722 OF BISMUTH AND THE NATURE OF "RAIES ULTIMES."

By A. L. NARAYAN and K. R. RAO.

Received March 3, 1926.

ABSTRACT.

Following the previous experiments of one of the authors on the nature of the green line λ5350 of thallium, a detailed examination of λ4722 of bismuth was made with a quartz Lummer plate, and it has been found that there is not even the slightest trace of absorption in the main component or in the satellites, even at 1,000° C. The fluorescence of the vapour under optical stimulation was studied, and it was found that when excited by radiation of wavelength from λ6000 to 3500 the fluorescence was a banded spectrum in the orange-yellow, while when excited by ultra-violet radiation from a bismuth arc, the fluorescence was bluish, and consisted of $\lambda\lambda$4722 and 3068, thereby showing that λ3068 represents the minimum excitation energy and corresponds to transition to the normal orbit.

A NUMBER of investigators have called attention to the great intensity of the line λ4722 in the emission spectrum of bismuth vapour. Foote and Mohler have recorded this as a " raie ultime " and resonance line. According to de Gramont also, the line λ4722 constitutes the " raie ultime." Generally, the raies ultimes are supposed to be the resonance lines and the first members of principal series. They therefore correspond to transitions to the lowest level of the atom, and should be absorbed by the non-luminous vapour of the metal. In the course of our experiments on the absorption of bismuth vapour it was found that the line was not absorbed by a column of the non-luminous vapour even at the highest temperature used, viz., 1,400° C. So far as the authors are aware, no investigator has recorded the absorption of this line. Experiments recently conducted in this laboratory on the structure and absorption of the green line λ5350 of thallium and published in the Proc. Roy. Soc., showed that this line was complex, and that at about 800° C. only the main component was absorbed, while the absorption of satellites commenced at higher temperatures. It was therefore thought that possibly in the case of λ4722 of bismuth a similar thing might happen.

In this connexion it will be remembered that numerous physicists have studied in recent years the structure of this radiation, notable among them being Gehrcke and Bayer, Takamine, Aronberg and Nagaoka and Suguira, and have called attention to the great complexity of this line.

With a view, therefore, to determine whether this line actually represents the transition to the normal state of the atom, it was decided to study at length the absorption of this line with instruments of high resolving power. Detailed examination of the absorption of this line was made by a quartz Lummer-Gehrcke plate of resolving power equal to 300,000, as a result of which it was found that even at 1,200° C. there was not the slightest absorption, either in the main component or the satellites, as is clear from Fig. 1.

In these experiments radiation from a vacuum arc lamp, the lower electrode of which was a carbon cup containing bismuth which the upper electrode was pure carbon, was examined by the L.G. plate before and after passing through a long column of bismuth vapour. Fig. 1 (*a* and *b*) are the Lummer patterns of this line before and after passage through the vapour at about 1,000° C.

In recent years Klein and Rosseland* have shown from thermo-dynamic considerations that by impacts of activated atoms with electrons, atoms and molecules, transformations can take place unaccompanied by radiation. Franck and Cario† have shown that this active energy may be related to a variety of energetic exchanges, and that the argument of Klein and Rosseland could be used to prove the reversibility of excitation by atomic impacts. This showed that excited Hg atoms colliding with neutral atoms of a vapour that does not absorb the effective radiation may excite the atoms of the vapour, causing them to emit fluorescence spectra, provided the excitation potential of the vapour is less than 4·9 volts (the energy equivalent of λ2536). Such activated fluorescence was used in recent years by Franck and Cario, Kofferman‡ and Donat§ for testing series relationships. Kofferman found that in the case of bismuth the lines $\lambda\lambda$4722 and 3068 were excited. In the experiments of these authors, as energy was communicated to the atoms of the gas in question by excited Hg atoms, it was not possible for them to find the minimum amount of energy required to excite the atom of the gas in question, and thus obtain information regarding the ground orbit of the atom. Experiments were therefore made by the authors to study the fluorescence of bismuth vapour when excited by different monochromatic radiation.

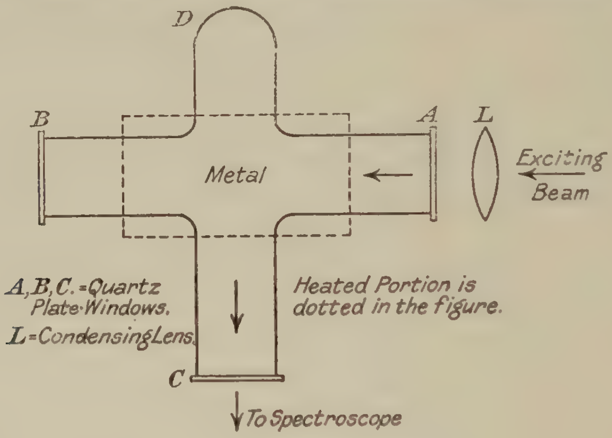
D
B
A
L
Metal
Exciting Beam
A,B,C.=Quartz Plate Windows.
L=Condensing Lens.
Heated Portion is dotted in the figure.
C
To Spectroscope

FIG. 4.

The vessel used for these experiments was in the form of a cross and made of seamless steel tubing 1 in. in diameter, having welded joints at the cross. A digrammatic sketch of the apparatus is given in Fig. 4. The end D is permanently closed with a welded joint so as to serve as a perfectly dark background.

The primary or exciting beam passes along AB, and is brought to a focus at the centre of the cross, and fluorescence was observed in the direction DC against the

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* Klein and Rosseland, Zeit. f. Phys., 4, 46 (1921).
† Franck and Cario, Zeit. f. Phys., 10, 185 (1922).
‡ Kofferman, Zeit. f. Phys., 21, 316 (1924).
§ K. Donat, Zeit. f. Phys., 19, 345 (1924).

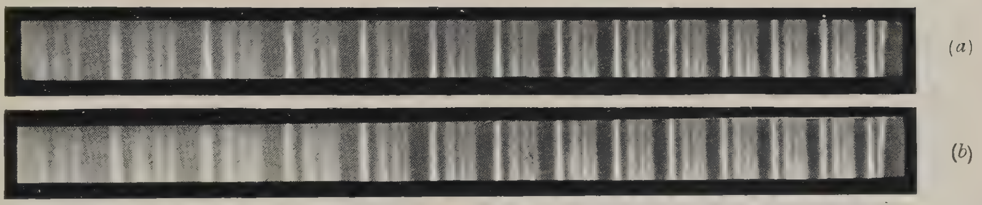

FIG. 1.

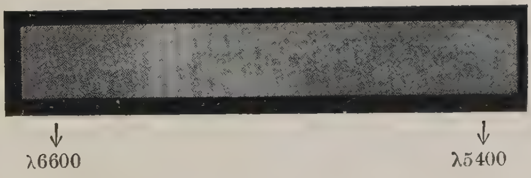

FIG. 2

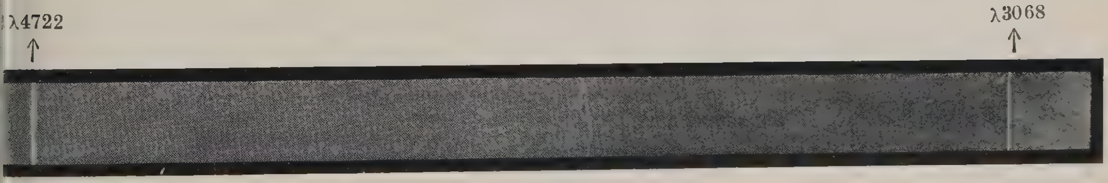

FIG. 3.

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black background. With a view to photograph even the faint fluorescent light, it became necessary to avoid all scattered light, and therefore between the slit of the observing spectrograph and the end C of the cross there were placed a number of co-axial diaphragms. The dotted portion was subjected to preliminary heating, and the tube was exhausted to a pressure of 1 mm. at 200° C., and then the temperature was raised to 550-600° C., when observations were made in the lateral direction by illuminating the vapour with different radiations, obtained by using different colour filters.

When excited by radiation of different wavelengths from λ6000 to λ3500, the fluorescence consisted of a banded emission, Fig. 2, in the orange-yellow, exactly corresponding to the banded spectrum emitted by the vapour under purely thermal excitation at about 1,300° to 1,400° C. recently photographed by one of the authors, the only difference between these two being that in temperature emission the bands are sharp. But when the vapour was excited by light from the bismuth arc, in which the radiation λ3068 was predominant, it was found that the fluorescent radiation was bluish, which was found to correspond to λ4722 and 3068; and, further, when the ultra-violet radiation from the arc was cut off by the interposition of a glass plate, the bluish fluorescence was suddenly extinguished, thereby showing clearly that λ3068 corresponds to the minimum excitation energy of the vapour and that this line corresponds to the transition of the valence electron to the normal orbit of the atom, while λ4722 corresponds to transition to an excited state. Further, in spite of the fact that the probability of transition to the normal orbit is necessarily greater than that to the excited state, and that, therefore, λ3068 should be more intense, this line is observed to be weaker than λ4722 in Fig. 3. This is probably due to the fact that λ3068 is reduced in intensity on account of absorption by the cooler vapour in the surrounding region.

These observations, together with those on the absorption of λ4722, clearly show that λ4722 corresponds to a transition to one of the excited states.

XXXII.—THE DISTRIBUTION OF INTENSITY IN A POSITIVE RAY SPECTRAL LINE.

By M. C. Johnson, *M.A.*, *M.Sc.*, Birmingham.

Received March 6, 1926.

(*Communicated by* Prof. S. W. J. Smith.)

Abstract.

In studying the intensity distribution in a spectral line emitted by positive rays, a line structure was found which is not familiar in ordinary discharge tube spectra. This structure would not appear under the low resolving powers necessitated by the extreme faintness of the light emitted in most investigations of Stark and Doppler effects in positive rays. The technique required to make possible the study of structure in such faint lines is discussed in detail. The explanation of the structure in terms of Doppler and Stark components is compared with the theory of broadened lines in ordinary discharge tube spectra.

Introduction.

The work of Wien and his pupils* has shown that when positive rays have passed through a perforated cathode the conditions under which they excite spectra are more simple and controllable than the spectral conditions of an ordinary discharge tube. On this account the spectroscopy of positive rays has been exhaustively studied by Stark, Wien and others for the splitting of lines in applied homogeneous electric fields, and for the shift of lines due to the comparatively homogeneous velocities of the ions in the positive ray stream. This work has, however, mostly been carried out with prismatic spectrographs. The present Paper describes an attempt to apply to positive rays the more minute examination of interference analysis, aided by the wedge method of studying intensity distributions in spectral lines, due to Merton. To eliminate the large Stark effects studied by other workers, the positive rays were allowed to stream into a chamber which was either made electrically force-free or could be subjected to small known potential differences. To eliminate large Doppler effects, the positive ray chamber was made narrow enough to act itself as a wide slit source of light, and the collimating axis of the optical train was kept perpendicular to the axis of the discharge tube.

I.

The light emitted in the positive ray chamber is very faint compared with usual sources of illumination in spectroscopy. The prismatic photographs of previous workers have required exposures of several hours. To attempt further a study of intensity distribution across a highly resolved line structure requires a combination to be made of several optimum conditions. Some of these conditions involve prolems about which evidence is conflicting or uncertain. We shall discuss the ollowing :—

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* Summarised in Handbuch der Radiologie, IV, 1 (1923). *See* also Proc. Phys. Soc., 37, 324 (1925).

(1). The design of tube and cathode to provide the greatest intensity of positive ay light.

(2). The control of the intensity ratio of the required lines to other associated pectra.

(3). The thermal and mechanical stability of the system for long exposures.

(4). The photographic sensitivity of plates for definite spectral regions.

(5). Use of the wedge and measurements required with it.

(1). Each of the successive discharge tubes used was made up of two pieces of lass tubing, about 30 cm. and 5 cm. long respectively, waxed on to opposite sides f a central disc which held the cathode, and which could be water cooled (Fig. 1).

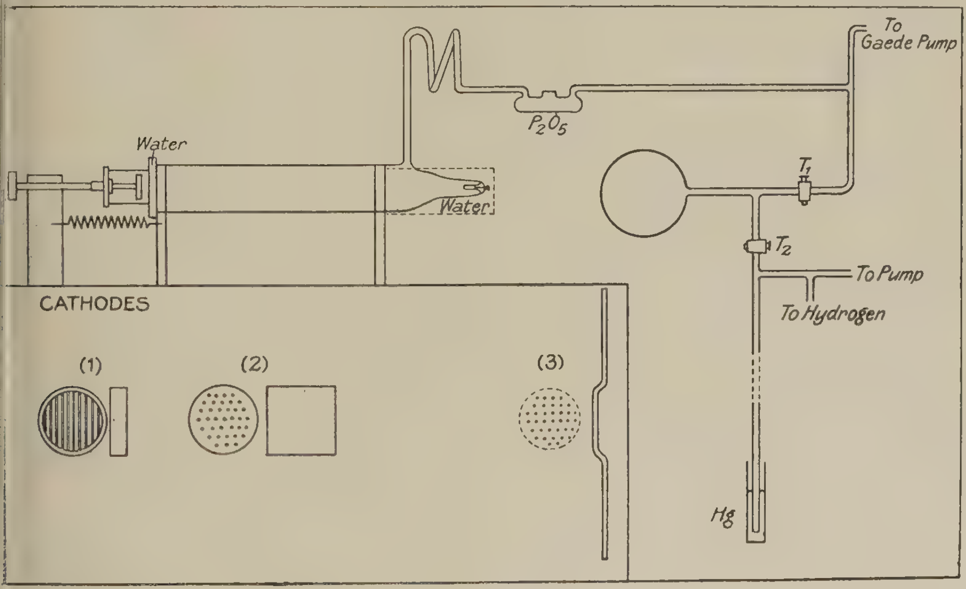

Fig. 1.

his general type of design was taken from the Shearer X-ray tube, and is an exception o the remark of Wien* that in England a spherical positive ray chamber is the rule. t has the advantage of allowing several cathodes in turn to be inserted in the central isc, and also of allowing a rigid auxiliary electrode to slide in and out of the brass nd piece of the chamber furthest from the anode. The anode end was drawn off nto a narrow tube for its electrode, as this causes the tube to be self-rectifying.† t was soon found that the diameter of the perforated cathode need not be much reater than 1·5 cm., as when the gas pressure is low enough to allow positive rays o develop, the stream becomes concentrated along the axis of the tube.

The thickness of the cathode and the size of perforations was found to be very important. Under many conditions a beam of positive rays in hydrogen extends luminous for several centimetres, until the pressure is below 0·03 mm. Hg.* Relying on this, one cathode (Fig. 1, inset 2) was made nearly 2 cm. thick, to enable the light behind to be observed clear of the central disc of the tube. The cathode, however, only allowed very faint luminosity in the positive ray chamber. A much thinner cathode was built up of aluminium blocks, separated at their ends by aluminium strips 0·1 mm. thick (Fig. 1, inset 1). This gave a greater intensity of positive ray light, in spite of the very narrow apertures. Since this increased intensity seemed traceable to decrease of thickness of cathode rather than to increase of perforation aperture, which was negligible, a final cathode was made as follows: A sheet of aluminium 3/32 in. thick was cut into a disc about 10 cm. diameter, and spun in a lathe while a blunt tool pressed out the central two centimetres. As many 1 mm. holes as possible were then drilled in this central portion. The two glass tubes were waxed on to opposite sides of this disc, the central portion projecting sufficiently (Fig. 1, inset 3) to allow a small angle of illumination to emerge from the positive ray chamber, and yet not projecting enough to encourage the positive rays in the main tube to diverge to the non-perforated edge of the disc. The parallelism of the back of the cathode with the auxiliary electrode was secured by accurate grinding of the edges of the shorter glass tube.

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* Proc. Phys. Soc., 37, 332.

† Aston, Isotopes, 48.

With this final form of tube, at 3 milliamps. discharge current, the positive ray glow between cathode and auxiliary electrode was a brilliant and perfectly homogeneous disc of light, 1·5 cm. diameter and 0·14 cm. thick, as bright as any part of the discharge tube. With hydrogen, its visual intensity rose as the gas pressure in the whole tube was decreased, had a maximum corresponding to a dark space of about 5 cm., and at still lower pressures became fainter.

(2). Hydrogen in a discharge tube can emit a many-lined spectrum and a continuous spectrum, as well as the Balmer series, with whose lines we are concerned. It is therefore necessary, in any work where economy of light is urgent, to investigate the control of the ratio of intensities in these spectra.

Since the many-lined spectrum is molecular and the Balmer spectrum atomic, some means must be found of suppressing the recombination of dissociated molecules. Alternatively, we must increase the exciting agency to allow predominance of those encounters which not only dissociate but ionise. According to R. W. Wood† and Merton,‡ water vapour brings out the Balmer series sharply, probably through the former alternative, by suppressing the catalysis of the glass surface. R. W. Wood increases excitation by the high current densities of a condensed discharge, which he finds improves the Balmer spectrum. This was tried in the present experiment but condensers in the circuit decreased the intensity of the positive rays, and slightly decreased the ratio of the Balmer intensity to the many-lined intensity. This i one of the many cases in which positive ray luminosity does not follow the same law as the ordinary discharge. The low gas pressures required for positive rays prohibited the use of water vapour to intensify the Balmer spectrum. G. P. Thomson states§ that in the positive ray spectrum of hydrogen the atomic lines are stronges relative to the molecular lines at the higher pressures; but his pressures are all very much lower than those in Wood's and Merton's ordinary tube spectra, where water vapour was successfully employed. Stark* insists on great purity of the gas for the strength of Balmer lines in positive rays. Having found the weakness of the analogy between ordinary spectra and positive ray spectra, this insistence was followed rather than the reliance on impurities advised by Wood and Merton. In the present work the hydrogen was taken from a commercial cylinder, and freedom from intermixture with air was secured by the following means (Fig. 1) :—

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* Dempster, Astroph. J., 57, 193 (1923).

† Phil. Mag., 42, 729 (1921), and 44, 538 (1922).

‡ Proc. Roy. Soc., A., 97, 307 (1920).

§ Phil. Mag., 40, 240 (1920).

The tap T_2 is closed and T_1 opened, while the whole system is evacuated to a cathode ray stage in air. T_1 is then closed. The lower tubes are then evacuated until mercury is high in the long vertical tube. Gas is then admitted until it drives the mercury down again, and bubbles out in the bottom vessel. This tube is then re-evacuated and hydrogen again admitted. This is done several times, until the atmospheric impurities in the hydrogen below T_2 must be extremely slight. T_2 is then opened, and the spherical reservoir filled at atmospheric pressure. T_2 is then closed, and the reservoir suffices for repeated washing out of the whole apparatus with hydrogen, and for several weeks' refillings of the discharge tube. Observations can then be made with always the same degree of gas purity.

Each time the apparatus was opened and air admitted during changes of cathode adjustment, etc., several days of repeated evacuation and washing out were required, owing to the large metal surfaces in the discharge tube, which required very extended treatment before they ceased to emit occluded gases under a heavy discharge. Stark's requirement of gas purity was confirmed, contrary to the experience with ordinary discharge tubes, the Balmer spectrum being very sensitive to the state of the metallic electrodes. If fresh hydrogen was admitted from the reservoir at any time, the intensity of the Balmer series did not at once increase, but only after an hour or two of discharge.

The relative intensity of Balmer to many-lined spectrum was examined throughout the tube at its best—i.e., when the apparatus had been sealed for some weeks. The ratio of these intensities was at its minimum near the anode, increased regularly along the tube up to the cathode face, and then suddenly increased again on the further side of the cathode. This observation is not in agreement with the common statement that the spectrum of the positive rays is the same as that of the cathode face.

The greatest intensity of the Balmer series was only reached after the apparatus had been sealed long enough for a film of sputtered metal to have covered the inside of the main tube. This probably acted as the water vapour is said to act in R. W. Wood's tubes—i.e., hindered the catalytic action of the glass in turning atomic hydrogen back into molecular, after dissociation in the discharge.

(3). Even with the maximum efficiency for the development of the Balmer lines by the above methods, the light in the positive ray chamber was far fainter than is usually accepted as a minimum for interferometer analysis. Exact focussing could only be done indirectly by illuminating the positive ray chamber from a brilliant external source of hydrogen radiation. Hence it was only possible by trial and error to ensure that the very narrow image of the chamber remained on the slit of the spectroscope during an exposure of several hours. The thermal and mechanical stability for this purpose of spectroscopes, wedge and camera were assured by building the apparatus up on stone piers, in an underground room with thick walls and no windows. The thermostatic properties of this room were tested to within 0·2°C. during the whole of the day. Night constancy was not so strictly guaranteed, and trouble invariably followed attempts to expose intermittently over more than a day.

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* Ann. der Ph., 21, 401 (1906).

The stability of the tube itself was attained by fixing the cathode disc rigidly, and allowing expansion to take place in other directions which did not matter. The ebonite support of the cathode disc (Fig. 1) was rigidly fixed to the tube, and was itself held by a strong spiral spring to a very massive iron pillar attached to a slate slab. A fine screw adjustment pressed through an insulator against the small end of the discharge tube; focussing could be corrected by small movements of this screw, which then kept the cathode region fixed for a number of hours. The warming of the tube, which even water cooling of anode and cathode did not quite prevent, could then be neglected as a source of movement.

(4). Authorities differ as to the colour sensitivity of photographic plates. The researches of the American Bureau of Standards* indicate that most panchromatic plates allow as great a darkening, for equal conditions, in the H_β region as in the H_α region; but personal opinions of several workers place the H_β region sensitivity low. In the present case the intensity distribution in the Balmer series of the positive rays was such that, visually, H_β was clear through prismatic spectroscope and an échelon in series, but not through the wedge. H_α was not seen at all. But photographs on Ilford's special rapid panchromatic plates showed a much stronger H_α than H_β. Accordingly, most of the measurements had to be confined to H_α. For this line, exposures were reduced by bathing the plates for four minutes in a solution of 1 c.c. ammonia in 100 c.c. water, then washing in alcohol, drying, and exposing as soon as quite dry and firm. This treatment, based on the work of the Bureau of Standards, was found to produce at best several hundred per cent. increase of sensitivity to H_α, but hardly any increase for H_β. Occasionally the treatment, though applied without any change of procedure, failed, and decreased the sensitivity. Another peculiarity noticed was that darkening of the image increased with time of exposure up to about four hours, but beyond that did not increase. The known non-linear relation of darkening to time does not seem adequate to explain this, considering the faintness of light and image.

(5). The light entering the slit of the optical train passed through a Hilger constant deviation prism spectroscope, by which one wavelength was focussed on the slit of a twenty-plate échelon grating. A camera was improvised for this, taking small plates for a single spectral line. In front of the slit of the prism spectroscope was a wedge, of the type used by Merton.† It consists of a wedge-shaped piece of neutral tinted glass, 7 mm. high, cemented to a wedge of clear glass, the whole forming a parallel sided plate whose optical density varies from 0·2 at the bottom to 4·2 at the top. A spectral line seen through this wedge under high resolving power appears broad at the bottom, where not only the bright core, but the faint wings of the line, are transmitted, and narrows upwards until the increasing density of the wedge absorbs, firstly, wings and finally the core of the line. From measurements of apparent widths at different heights in the line, the intensity distribution across different forms of the same line radiation can be compared. For this measurement several instruments were used, a choice of magnifying powers being needed to bring out different features. A Hilger photo-measuring micrometer was compared with a Watson micrometer and microscope, fitted with an accurate two dimensional travelling stage. A Poynting parallel plate micrometer was also used for certain measurements.

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* Scientific Papers, No. 422.

† Proc. Roy. Soc., A., 92, 322 (1916).

Owing to the curvature of all lines from a prismatic spectroscope, even illumination in the échelon image could only be obtained by careful adjustment of the relative apertures of the two slits. This had to be at the expense of the illumination of the lowest end of the wedge. Accordingly, all spectral lines had to be measured from the broadest part of the line upwards, and the falling off of intensity at the bottom of the line neglected.

II.

Photographs of H_α of the positive rays through prism and échelon showed always a broadened line. This was at first put down to movement, mechanical or thermal, during the long exposures with the earlier design of cathode, which sometimes amounted to nine hours. The precautions described in I(3) were adopted to eliminate this possibility. The broadening persisted, however, and it was not till the wedge was used that a definite reason for it could be assigned. To test the possibility of broadening being an instrumental defect not remedied by these arrangements, photographs were taken with very long exposures, but lower resolving power. The sharpness and fineness of the results showed that if a radiation was nearly monochromatic there was nothing in the apparatus to prevent it appearing so on the plate. This test also eliminated the possibility of the broadening being due to some unknown type of spreading of the image, affecting faint lines long exposed.

The broadening was thus accepted as characteristic of the positive ray light under high resolving power, and compared with the broadening of a line in ordinary tube spectra. Condensed discharges and uncondensed discharges were photographed with the échelon and wedge under the same conditions as the positive ray photographs. This was ensured by allowing them to illuminate the positive ray chamber itself from behind, and photographing the spectrum of the chamber with the same adjustments as when it was filled with the positive ray glow. Fig. 2 shows the exact intensity distribution across each line, measured by the methods described in I(5). The inset is a drawing of the general appearance of the three types of line. The ordinary tube spectra have widths in each case fading perfectly regularly from bottom to top of the wedge, that of the condensed discharge (C) obeying the law of that type of intensity distribution, and the uncondensed discharge (B) obeying its law ; though in this case the difference between the two types is not large, as the condensed broadening had to be slight enough not to overpass the order separation of the échelon. The positive ray line (A), on the other hand, consists of a short dark region, representing a line almost as broad as that of the uncondensed discharge, but of only a small fraction of the latter's intensity, and hence broadened on a far more extensive scale. In contrast to this squat image is a faint central line, extending about three times the length of the former. This is very clear in all the negatives, but even in the best prints is hardly visible, on account of its faintness and fineness. Its different origin and constitution from that of the short broad line appears very strikingly under the microscopes, where high magnification causes it to disappear, leaving the whole image constituted by the broad portion alone.

III.

The measurements of Fig. 2 were calibrated by measuring the separation of orders in the échelon spectrum, the constants of which were calculated in a previous Paper.* The positive ray line has an apparent width of just over 0·5 A.U. at the base of the wedge. The possible reasons for the broadening of a line are as follows :—

(*a*) The random Stark effect of the intermolecular fields. This produces a symmetrical broadening in a line, such as those of hydrogen, whose Stark components are symmetrical. It occurs at high pressures or in condensed discharges, and is seen in its incipient stage in Fig. (*C*2). The continuous intensity curve is due to the random directions and magnitudes of the intermolecular fields. It is clear

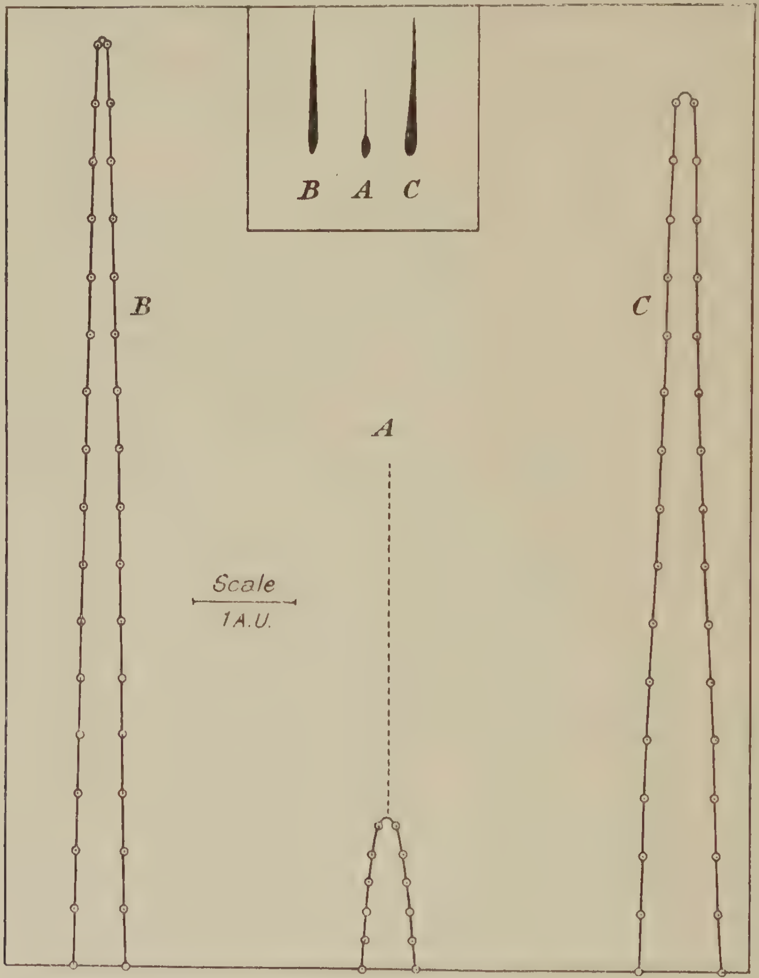

Fig. 2.

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* Proc. Phys. Soc., 37, 249 (1925).

that this cannot account for the positive ray line, whose amount of broadening relative to central intensity is so much greater as to imply, on such an explanation, a density of charged atoms of a different order from any met with in these experiments. Nor would it account for the distinction of the short broad portion from the central line in the wedge image.

(*b*) The random Doppler effect due to the velocity of thermal agitation produces a broadening proportional to the square root of the temperature. That this is not the cause of the positive ray broadened line is clear from the fact that the positive ray chamber remained throughout very much cooler than the tube used for the uncondensed discharge, in which the random Doppler effect is the main source of breadth.

(*c*) In the positive ray chamber the direct Stark effect due to a unidirectional applied field would produce a broadening, if not sufficient to split the line into visibly separated components. To avoid the possibility of any such difference of potential being maintained between the cathode and auxiliary electrode, due to the latter charging up in some way, the two were in metallic connexion throughout these photographs.

(*d*) There remains the possibility of the broadening being due to a unidirectional Doppler effect—i.e., to the velocity of the ions moving towards the cathode in the potential fall of the dark space, and continuing their flight behind the cathode. Velocities up to 10^8 cm. per sec. have been observed in the positive rays when the slit of the spectroscope has been along the axis of the tube. To avoid these effects in the present case, when fine lines were desired, the axis of the optical train was perpendicular to the line of motion of the ions. It is possible, however,* that the slight unavoidable component of positive ion velocity allowed by the divergence of the beam from the chamber would be sufficient to account for the broadening.

The velocity v of a particle emitting light is related to the Doppler change of wavelength and the velocity of light c by the equation

$$d\lambda = \lambda \frac{v}{c}$$

while the velocity of the positive rays is related to the charge e and the mass m of the positive ion, and the potential V through which it has fallen in the dark space by the equation

$$v = \sqrt{\frac{2e}{m}V}$$

Now Wilsar has shown† that the Doppler shift of the spectral line from positive rays in certain gases does not increase indefinitely with the potential, but reaches a limit for a potential characteristic of the gas. This highest "effective" potential in hydrogen for producing Doppler effects is about 4,600 volts. Owing to the uncertainty as to whether all, or only the greater part, of the fall of potential across the discharge tube occurs in the dark space, it is not possible to know the exact potential through which the positive ions have fallen in the present case. It is certainly more than 4,600 volts. Since the charge and mass of the hydrogen positive ion are known, it can be shown from the above equations that to reduce the displacement for the maximum effective fall of potential in hydrogen, to the half width of the line observed in the present photographs, and interpreted as a Doppler effect, would require a numerical factor of about 70. This reduction would be effected by the sine of an angle of about 50′. If, then, some of the light which enters the slit of the spectroscope emerges from the positive ray chamber one degree on each side of the normal to the axis of the discharge tube, there will be a sufficiently large component of the velocity of the positive rays to produce a Doppler effect in the spectrum of the magnitude of the broadening measured. Even with a small aperture lens system, and the very narrow positive ray chamber used, this divergence must be reached. This explanation accounts for the symmetry of the broadening, since the divergence of the light introduces components both with and against the stream.

---

* I am indebted to a conversation with Mr. P. M. S. Blackett for this suggestion.

† Ann. der Ph., 39, 1251 (1912).

The explanation also accounts as follows for the faint " tail " seen in all the positive ray photographs. The positive ray chamber is known to produce two spectral lines in general for each monochromatic radiation, a line displaced by the Doppler effect, and an undisplaced line (Stark's " resting spectrum "). Wilsar showed* that the carriers of the displaced lines are the high velocity particles themselves, which come from the discharge tube and are stimulated to emit light by collisions with the gas in the positive ray chamber. Dempster† showed that the displaced spectrum is in some cases due to particles whose duration of luminosity has lasted during their flight since they became stimulated in the discharge tube itself. In both these cases, if photographs are taken along the axis of the tube, the Doppler effect is shown at its maximum, and the line appears to the violet side of its normal position. The " resting spectrum," on the other hand, is not subject to a Doppler effect, since it is emitted by the comparatively stationary atoms, which were already in the positive ray chamber, and are stimulated by the impact of the arriving positive ions.

The short and broadened part of the positive ray line in Fig. 2 is thus most probably due to small components in both directions of the moving spectrum, and the " tail," or central undisplaced line, is due to the " resting spectrum."

IV.

For comparison with the above results, small known potentials were applied between the cathode and auxiliary electrode, to find whether direct Stark effects could be detected superposed on the Doppler structure. For H_α no effect was detected with the small fields which could be maintained across the gap, whose conductivity could not be decreased without decreasing its luminosity. For H_β the displacement of Stark components is known to be greater than for H_α, and a direct Stark effect was less difficult to produce and measure. Unfortunately, the special hypersensitising methods described in I(1) are not applicable to the H_β region, and, even with the longest exposures, no satisfactory photographs were obtained through the Merton wedge. A plate of H_β, taken with échelon but without wedge, was measured. This contained two images, one taken with cathode and auxiliary electrode connected together, and one with a field of 1,700 volts per cm. applied

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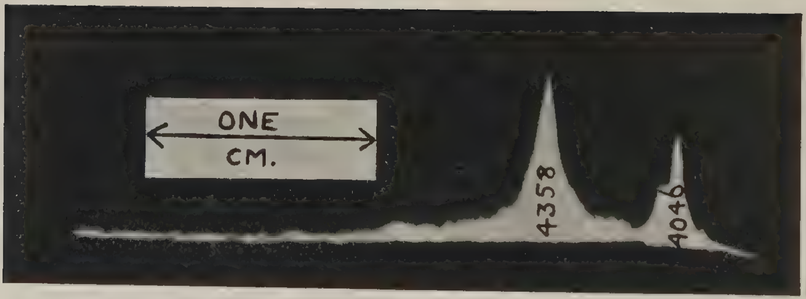
* Phys. Zeitshr., 12, 1091 (1911).

† Astroph. J., 57, 193 (1923).



"EDGEWISE" SPECTRAL PHOTOGRAPH.

[To face page 333

between them from a battery of accumulators. To eliminate the difficulty of even illumination caused by the curvature of the prismatic image, a corresponding point on each line was selected. The apparent widths, in the arbitrary units of the Hilger measuring instrument, were as follows :—

Without field : 17·4 16·5 17·1.
With field : 22·4 22·9 22·1.

In the arbitrary units of a Poynting parallel plate micrometer, the widths were found to be :—

Without field : 74 75 78.
With field : 109 111 109.

The ratio of the mean widths, with the two measuring instruments, was 1·3 and 1·4 respectively.

Owing to the uncertainties attendant on measuring apparent widths of lines—i.e., on measuring widths without knowing the intensity distribution—the results cannot be used in the problem of deciding the absolute value of the Stark displacements in very small fields. They are of interest as involving probably the smallest potential at which a Stark effect has been seen, and as comparing, in the same highly resolved spectrum, the orders of magnitude of direct Stark and Doppler effects, due to homogeneous fields and homogeneous velocities. Both these effects are only observable in positive ray spectra, and may be contrasted with the familiar indirect, or random, Stark and Doppler effects due to molecular fields and molecular agitation respectively, which determine the breadth of ordinary tube spectra.

It is a pleasure to thank Prof. S. W. J. Smith, F.R.S., for his kindly encouragement, and for the generous facilities he has provided for this work. I also owe a very great deal of instrument construction and help to Mr. G. O. Harrison.

DISCUSSION.

Mr. J. GUILD : There is just one practical point in connection with the wedge-method of spectrophotometry to which I would like to drawn the author's attention. It is probably not of serious consequence in work which is of a semi-qualitative character, but is of great importance if any attempt is made to derive accurate intensity information from wedge photographs. In most spectrographs the length of the slit is a fairly large proportion of the height of the various stops—diaphragms, prism boundary, etc.—which limit the pencil of rays reaching any part of the plate. In a Paper at the recent Optical Convention dealing with colorimetry and including spectrophotometry, I pointed out how serious errors may arise due to this fact. The difficulty could be avoided by using a specially designed instrument, or by inserting suitable stops in appropriate positions in existing instruments. Unless precautions of this character are taken, however, very large errors may be obtained.

Dr. L. F. RICHARDSON said that the difficulty mentioned by Mr. Guild might perhaps be overcome by omitting the grey wedge and placing the photographic film so that the light moved parallel to the plane of the film and struck one edge of the film nearly at right angles, the edge of the film passing through the centre of each of the spectral images of the slit. He had recently taken such an " edgeways " photograph of the bright spectrum of mercury vapour and had found that the relative photographic effects of the different spectral lines were very clearly shown : (i) By the relative depths to which light penetrated the film amounting to 1·5 cm. for the line 4358 Å ; and (ii) by the relative widths of the broadening, due probably to halation, near the edge where the light entered. To protect the broad faces of the film they were covered with black paper. To keep the film in a plane during exposure it was pressed between slabs of plate glass. The figure shows the result, except for much detail lost in reproduction. The exposure was 25 minutes. Ilford " double-coated X-ray film " was used. The rim of the lens which focussed the light, subtended an angle of 0·12 radian at the edge of the film, which was placed 0·4 cm. nearer the lens than the focal plane in air.

AUTHOR'S reply : With regard to Prof. O. W. Richardson's suggestion of comparing the shape of the undisplaced line in the longitudinal effect with the transverse spectrum, the échelon measurements of the latter could not be repeated with the former on account of the too great breadth of the longitudinal Doppler effect for interferometer work. With regard to his question as to the character of the many-lined spectrum in the positive rays, I can only say it is always fainter relative to the Balmer intensity than in the other parts of the discharge.

I am very grateful for Mr. Guild's remarks on the accuracy of the photometric wedge method, and for Mr. L. F. Richardson's suggestion of an important alternative method. The difficulties mentioned by Mr. Guild were increased by the curvature of the spectral line from a prism, and my only check on the accuracy was the comparison photographs without the wedge, taken to ensure at least an apparently even illumination of the whole length of the line.

DEMONSTRATION OF SOME SIMPLE EXPERIMENTS WITH THERMIONIC VALVES.

By E. H. Rayner, *Sc.D.*

A METAL plate or wire, connected to the grid of an ordinary receiving valve operating with 20 to 50 volts on the anode, acts as a sensitive indicator of the presence of an electric field in the neighbourhood. A piece of ebonite or sealing wax electrified by friction will cause an appreciable change in the anode current, when brought within 3 ft. or more of the conductor connected to the grid. A unipivot micro-ammeter or a reflecting galvanometer suitably shunted forms a satisfactory indicator. If such a negatively electrified object as ebonite is brought close to the grid conductor, the anode current may be reduced practically to zero, a positive charge being attracted to the near parts of the conductor and a negative charge being repelled to the grid. If the rod is quickly removed the anode current may flow momentarily, but cease again for several seconds, then gradually rise to the normal value. This does not happen with all valves, and it may be affected by the degree of insulation of the valve cap. If the electrified rod is removed somewhat more slowly the same valve may not show this second reduction of the anode current to zero, either effect being produced at will.

If a square piece of sealing wax is electrified and placed on a metal plate about 3 in. square connected to the grid, the anode current will be reduced to zero, with a suitable valve and anode voltage. It will gradually rise again to its normal value, which can again be reduced by turning the sealing wax so that another of the four sides touches the plate. Not only can this be done once with each of the four sides, but possibly several times over, the reason apparently being that the wax only touches the plate in a few points and these parts of it only are discharged by contact, and they become again charged by surface leakage from surrounding parts while the other three sides are being experimented upon. This effect was found to work well with a dull emitter valve which would not show the double reduction to zero of the anode current mentioned above.

In order to illustrate possible applications of induced grid electrification, a pendulum with a metal plate about 3 in. square was made to swing over a fixed similar plate with about a tenth of an inch clearance. The swinging plate was electrified by a battery of about 25 to 70 volts, one end of which was connected to the filament of the valve. A reversing switch was placed in the battery circuit, so that the pendulum could be either positively or negatively electrified. As the pendulum swings past the fixed plate, it affects the potential of the grid and increases or decreases the anode current twice every complete swing, according to the direction of the electrification. By suitably adjusting the conditions a variation of at least 2 : 1 in the current can be obtained. It sometimes happens that the first swing cuts the current down to zero for several seconds. It is not necessary to use such a large condenser as plates of the size mentioned. A horizontal piece of wire 4 in.

long projecting from the pendulum at right angles to the plane of its swing and passing over a similar fixed wire connected to the grid, with a clearance of a tenth of an inch or less, can be made to show quite an appreciable effect when electrified to 100-200 volts.

In general the use of a valve as an indicator of small relative displacements of metal surfaces which can be insulated from one another and charged to different potentials, seems to be one of some promise. When used with movements of the order of 1 or less periods a second, leakage in the valve cap becomes an important factor, and experiments of the type indicated are distinctly critical in showing up differences between valves.

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CONTENTS.

PAGE

XXVIII. Obliquity Corrections in Radium Estimation. By Ivor Backhurst, M.Sc. 277

XXIX. Viscosity of Water at Low Rates of Flow Determined Comparatively by a Method of Thermal Convection. By Albert Griffiths, D.Sc., and P. C. Vincent, B.Sc. 291

XXX. The Properties of Mutual Inductance Standards at Telephonic Frequencies. By L. Hartshorn, A.R.C.S., B.Sc., D.I.C. 302

XXXI. A Note on λ4722 of Bismuth and the Nature of "Raies Ultimes." By A. L. Narayan and K. R. Rao 321

XXXII. The Distribution of Intensity in a Positive Ray Spectral Line. By M. C. Johnson, M.A., M.Sc. 324

Demonstration of Some Simple Experiments with Thermionic Valves. By E. H. Rayner, Sc.D. 335